**Rules for Probability**

**Rule 1:** \[0 \leq P(A) \leq 1\]

**Rule 2:** With \(S = \text{sample space}\), \(P(S) = 1\)

**Rule 3:** In the case of equally likely outcomes,
\[
P(A) = \frac{\text{number of outcomes corresponding to event } A}{\text{total number of outcomes in sample space } S}
\]

**Rule 4:** Complement rule \(P(A^c) = 1 - P(A)\)

**Rule 5:** Disjoint events (aka mutually exclusive) can never be independent because if one event occurs the other event can’t.

**Rule 6:** General addition rule
\[
P(A \cup B) = P(A) + P(B) - P(A \cap B), \text{ or } \]
\[
P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)
\]

**Rule 7:** Events \(A\) and \(B\) are independent if knowing that one event occurs does not influence the probability that the other event occurs.

**Rule 8:** General multiplication rule
\[
P(A \cap B) = P(A \text{ and } B) = P(B \mid A) \times P(A), \text{ where } P(B \mid A) \text{ is a conditional probability}
\]

Special case: Multiplication rule for independent events.
\[
P(A \cap B) = P(A \text{ and } B) = P(A) \times P(B)
\]

which implies that \(P(A \mid B) = P(A)\) and \(P(B \mid A) = P(B)\)

Comments: Two events are independent when the occurrence of one event does not change the probability of the other event occurring. If \(A\) and \(B\) are independent, then having information for event \(A\) tells us nothing about event \(B\). Disjoint events are always dependent because if one event occurs the other event can’t (think about \(P(A \cap B) = 0\), but \(P(A) \times P(B) \neq 0\)).
Test for Independence of Events $A$ and $B$:

Check for $P(B|A) = P(B)$;

Check for $P(A|B) = P(A)$;

Check for $P(A \cap B) = P(A) \times P(B)$;

Otherwise, if dependent it will satisfy $P(A \cap B) = P(A) \times P(B|A)$

Test for Disjoint Events $A$ and $B$:

Check for $P(A \cap B) = 0$;

Check if events $A$ and $B$ overlap in the sample space $S$;

Check if events $A$ and $B$ can occur simultaneously (physical constraints determine this).