Problem 1  Tone-Modulated FM Signal  (12 points)

A 100 MHz carrier wave has a peak amplitude of 3.75 volts. The carrier is frequency modulated (FM) by a 3 kHz modulating tone such that the frequency deviation is 75 kHz. You are told that the modulated waveform passes through zero and is increasing in amplitude at time \( t = 0 \). Write the equation for this tone-modulated carrier waveform.

Problem 2  Bandwidth of a Tone-Modulated FM Signal  (12 points)

A carrier is modulated by a sinusoidal modulating signal \( m(t) \) of frequency 2 kHz, resulting in a frequency deviation \( \Delta f \) of 5 kHz.

(a) Find the bandwidth occupied by the FM waveform.
(b) The amplitude of the modulating sinusoid is increased by a factor of three and its frequency is lowered by 1 kHz. Find the new bandwidth.

Problem 3  Frequency Swing of an FM Signal  (12 points)

In an FM system, a 7 kHz modulating (or baseband) signal modulates a 107.6 MHz carrier wave such that the frequency deviation $\Delta f$ is 50 kHz.

(a) Find the carrier frequency swing of the FM signal and the modulation index $\beta$.

(b) Find the highest and the lowest frequencies exhibited by the FM signal.
Problem 4  Frequency Swing of an FM Signal  (12 points)

(a) Determine the frequency deviation $\Delta f$ and the carrier swing for an FM signal with a carrier frequency of 100 MHz and whose upper frequency swing is 100.007 MHz when modulated by modulating signal $m(t)$.

(b) Also find the lowest frequency swing of this FM signal.

Problem 5  FM Signal Parameters  (12 points)

An audio signal, baseband from 200 Hz to 4 kHz, frequency modulates (FM) a carrier of frequency $f_C = 50$ MHz. The frequency deviation per volt is 10 kHz per volt and the maximum audio voltage modulating it is 3 volts. Calculate both the frequency deviation $\Delta f$ and the bandwidth $BW$ of the FM signal.
Problem 6  FM Waveform  (12 points)

The figure below shows an FM carrier modulated by a single tone sinusoidal wave. Calculate both the carrier frequency \(f_c\) and the frequency of the tone frequency \(f_m\). Express both frequencies in kilohertz (kHz).
Problem 7  FM Differentialtor Demodulator  (28 points)

We know that FM signals are generally of the form,

\[
\phi_{FM}(t) = A_c \cos \left[ \omega_c t + k_f \int_{-\infty}^{t} m(\lambda) d\lambda \right]
\]

where the symbols have their standard meanings as used in our FM lectures. In Section 4.5 (starting on page 189) we know that one category of FM detector is the “differentiator demodulator.”

(a) Show that time differentiation of \( \phi_{FM}(t) \) yields a form that you can use to extract the baseband signal \( m(t) \) using an envelope detector (or equivalent).

(b) Consider the circuit shown below. It is a simple RC network as drawn followed by an envelope detector (as we studied when covering amplitude modulation). Derive the transfer function \( H(j\omega) \) for the stand-alone RC network. Remember \( H(j\omega) \) is defined in the sinusoidal steady-state as the output voltage divided by the input voltage. Let \( \omega_H \) be the 3-dB corner frequency which is equal to \( 1/RC \) as usual.

![Circuit Diagram](image)
(c) Show that the above RC network approximates (or functions as) a differentiator under the right conditions.

(d) Suppose the RC network from part (b) above has a 50-ohm resistor; we want to find the value of capacitor C under the following conditions. First, FM stations have carrier frequencies nominally around 100 MHz and we choose \( \omega_H \) to be 10 times greater than this nominal frequency of \( f_C = 100 \text{ MHz} \) (where \( \omega_C = 2\pi \times 10^8 \text{ Hz} \)). What is the value of capacitor C meeting these conditions?

**Extra Credit** (up to extra 10 points)

Continuing with Problem 7 – give one disadvantage of the RC network differentiator as considered in Problem 7.