Problem 1  Sensitivity of FM Slope Detector  (30 points)

One of the FM discriminators (FM detectors) we covered in our lectures on FM was the FM slope detector (see slide 79 in the lecture notes on Angle Modulation). The circuit schematic below shows a parallel RLC resonant circuit we shall use in this problem to study the sensitivity of a slope detector.

Assume the incoming FM signal is modeled as a current source (say the current from a receiving antenna). Of course, the sensitivity of this slope detector (used as a frequency to amplitude converter) depends upon the operating frequency – that is the offset between the carrier frequency $f_C$ of the FM signal and the network’s resonant frequency $f_0$.

The input current flows into the input impedance $Z(j\omega)$ of the resonant circuit where the impedance is given by the expression,

$$|Z(j\omega)| = \frac{R}{\sqrt{1 + Q^2 \left[ \frac{f}{f_0} - \frac{f_0}{f} \right]^2}}$$

where $Q = 2\pi f_0 RC = \frac{R}{2\pi f_0 L}$ and $(2\pi f_0)^2 = \frac{1}{LC}$.
In this problem we want to estimate the sensitivity of the slope detector in converting frequency variations (in Hz) to amplitude variations (in volts). Given an FM slope detector with $Q = 10$, resistance $R = 500$ ohms, and resonant frequency $f_0$, estimate the detector’s sensitivity. To do this we choose to operate the slope detector at the carrier frequency $f_C$ that is below the resonant frequency by approximately 10%.

One way to estimate the sensitivity is to evaluate the slope detector at two closely spaced frequencies, say at $f_C + \Delta f$ and $f_C - \Delta f$, with the frequency deviation $\Delta f$ being small. To do this choose $f_C + \Delta f = 0.91 f_0$ and $f_C - \Delta f = 0.89 f_0$. Thus, $2\Delta f$ is the change in frequency to be used in our calculation.

(a) With $Q = 10$ and $R = 500$ ohms, find the max and min values of the output voltage given a peak drive current $I_{FM}$ equal to 10 mA. This will allow you to determine the change in voltage for a change in frequency. We assume here that $f_C$ is nominally 90% of $f_0$. The sensitivity is then expressed as the difference in output voltage ($= I_{FM} \times |Z(j\omega)|$). Express your answer for sensitivity $S$ in volts per MHz.
(b) What is the resonant frequency $f_0$ of the slope detector?

(c) What is the bandwidth $BW$ of the resonant circuit for $Q = 10$?

**Problem 2  Pulse Modulation Possibilities**  (30 points)

We learned that with a sinusoidal carrier wave there were three possible parameters we could modulate to carry information. Now we want to consider communication using a series of pulses (i.e., rectangular waveforms).

(a) What pulse waveform parameters can we modulate to carry information in a communication system? [Hint: There are three.]

(b) Given the modulating signal $m(t)$ shown below, sketch an example of how each of the three pulse parameters can be used to represent the information contained in modulating signal $m(t)$. Graphically show how each parameter is used to represent signal $m(t)$ for the three cases you identified in part (a) above.
Pulse ________________ Modulation:

\[ m(t) \]

t

Pulse ________________ Modulation:

\[ m(t) \]

t

Pulse ________________ Modulation:
Problem 3 Nyquist Rate  (10 points)

A signal is band-limited with a bandwidth of $B = 3,200$ Hz. Suppose we require a guard band of 750 Hz; what should the sampling rate be to meet this requirement?
Problem 4 Sampling  (10 points)

A signal $m(t)$ with a bandwidth of 40 Hz is sampled at 25% above the Nyquist rate. Suppose the resulting sample values are found to be

$$
m(nT_S) = -1, \quad \text{if } -6 \leq n \leq 2, \quad \text{and}
$$

$$
m(nT_S) = +1, \quad \text{if } 3 \leq n \leq 5, \quad \text{and}
$$

$$
m(nT_S) = 0, \quad \text{otherwise}.
$$

What is the value of $m(nT_S)$ at $t = 0.0375$ second?

Problem 5 Pulse Code Modulation  (20 points)

Explain how pulse code modulation (PCM) differs from the pulse modulation schemes you identified in Problem 2.