Problem 1  Sensitivity of FM Slope Detector  (30 points)

One of the FM discriminators (detectors) we covered in our lectures on FM was the FM slope detector (slide 79 in the lecture on Angle Modulation). The circuit schematic below shows a parallel $RLC$ resonant circuit we use in this problem to illustrate the sensitivity of a slope detector.

Assume the incoming FM signal is modeled as a current source (say the current from an antenna). Of course, the sensitivity of this slope detector (used as a frequency to amplitude converter) depends upon the operating frequency – that is the offset between the carrier frequency $f_c$ of the FM signal and the resonant frequency $f_0$.

The input current flows into the input impedance $Z(j\omega)$ of the resonant circuit which is given by the expression,

$$|Z(j\omega)| = \frac{R}{\sqrt{1 + Q^2 \left[ \frac{f}{f_0} - \frac{f_0}{f} \right]^2}}$$

where $Q = \frac{2\pi f_0 R C}{2\pi f_0 L} = \frac{R}{f_0 L}$ and $(2\pi f_0)^2 = \frac{1}{LC}$

In this problem we want to estimate the sensitivity of the slope detector in converting frequency variation (in Hz) into amplitude variation (in volts).
Given an FM slope detector with \( Q = 10 \), resistance \( R = 500 \) ohms, and resonant frequency \( f_0 \), estimate the detector’s sensitivity. To do this we choose to operate the slope detector at the carrier frequency \( f_c \) that is about 90% of the resonant frequency.

A way is to estimate the sensitivity is to evaluate the slope detector at two closely spaced frequencies, say at \( f_c + \Delta f \) and \( f_c - \Delta f \), with the frequency deviation \( \Delta f \) being small. To do this choose \( f_c + \Delta f = 0.91 f_0 \) and \( f_c - \Delta f = 0.89 f_0 \). Thus, \( 2\Delta f \) is the change in frequency to be used in our calculation.

(a) With \( Q = 10 \) and \( R = 500 \) ohms, find the max and min values of the output voltage given a peak drive current \( I_{FM} \) equal to 10 mA. This will allow you to determine the change in voltage for a change in frequency. We assume here that \( f_c \) is nominally 90% of \( f_0 \). The sensitivity is then expressed as the difference in output voltage (= \( I_{FM} \times |Z(j\omega)| \)). Express your answer for sensitivity \( S \) in volts per MHz.

Solution:
We know that \( Q = 10 \), \( R = 500 \) ohms and \( f_C = 100 \) MHz. Using the equation for \( |Z(j\omega)| \) allows us to calculate the change in impedance for the two frequency points \( f_C + \Delta f = 0.91 f_0 \) and \( f_C - \Delta f = 0.89 f_0 \).

| \( f \)  | \( \left[ \frac{f}{f_0} - \frac{f_0}{f} \right] \) | \( \left[ \frac{f}{f_0} - \frac{f_0}{f} \right]^2 \) | \( \frac{|Z(j\omega)|}{R} = \frac{1}{\sqrt{1+Q^2\left[ \frac{f}{f_0} - \frac{f_0}{f} \right]^2}} \) | Voltage |
|-------|-----------------|-----------------|-----------------|--------|
| \( f = 0.89f_0 \) | - 0.2356 | 0.05457 | 196.8 \( \Omega \) | 1.968 V |
| \( f = 0.91f_0 \) | - 0.1889 | 0.03568 | 233.9 \( \Omega \) | 2.339 V |

The voltage variation over approximately 2\% of \( f_0 \) is \((2.339 \text{ V} - 1.968 \text{ V} = 0.371 \text{ V})\). We can determine the frequency range from

\[
\frac{f_C + \Delta f}{0.91} = f_0 = \frac{f_C - \Delta f}{0.89} \quad \text{or} \quad (-0.02247) f_C = -(2.02247) \Delta f
\]

which gives \( \Delta f = 0.011111 \cdot f_C = 1.111 \text{ MHz} \)

The range for the frequency is \( 2\Delta f = 2.222 \text{ MHz} \) and therefore the sensitivity \( S \) is

\[
S = (0.371 \text{ Volt})/(2.222 \text{ MHz}) = 0.167 \text{ Volts/MHz} \quad \Leftarrow
\]

(b) What is the resonant frequency \( f_0 \) of the slope detector?

Answer:

Using

\[
\frac{f_C + \Delta f}{0.91} = f_0 \; ; \quad \text{With} \; \Delta f = 1.111 \text{ MHz and} \; f_C = 100 \text{ MHz},
\]

\[
f_0 = 111.1 \text{ MHz} \quad \Leftarrow
\]

We are approximately 11\% below the resonance frequency as we are using the \( RLC \) circuit as a slope detector.
(c) What is the bandwidth $BW$ of the resonant circuit with a $Q = 10$?

We can calculate the bandwidth $BW$ from the simple expression of

$$BW = \omega_U - \omega_L = \frac{\omega_0}{Q} = \frac{111.0 \text{ MHz}}{10} = 11.11 \text{ MHz}$$

Problem 2  Pulse Modulation Possibilities  (30 points)

We learned that with a sinusoidal carrier wave there were three possible parameters we could modulate to carry information. Now we want to consider communication using sequences of pulses (i.e., rectangular waveforms).

(a) What pulse waveform parameters can we modulate to carry information in a communication system?

Answer: 1. Amplitude
        2. Width (or duration), and
        3. Position

(b) Given the modulating signal $m(t)$ as shown below, sketch an example of how each of the three pulse parameters could be used to represent the information contained in modulating signal $m(t)$. Sketch how each parameter is used to represent signal $m(t)$ for the three cases you identified in part (a) above.
Pulse ________________ Modulation:

\[ m(t) \]

\[ t \]

Pulse ________________ Modulation:

\[ m(t) \]

\[ t \]
Pulse Modulation:

\[ m(t) \]
Answers:

Pulse Amplitude Modulation (PAM)

Pulse Width Modulation (PWM) or Pulse Duration Modulation (PDM)
Problem 3  Nyquist Rate  (10 points)

A signal is band-limited with a bandwidth of $B = 3,200$ Hz. Suppose we require a guard band of 750 Hz; what should the sampling rate be to meet this requirement?

Answer:

The Nyquist theorem requires for a signal of bandwidth $= 3,200$ Hz that the sampling rate must be at least twice the bandwidth. However, we are required to add a guard band of 750 Hz beyond the Nyquist rate.

Therefore, the sampling rate $R$ is

$$R = 2 \times (3,200 \text{ Hz}) + 750 \text{ Hz} = 7,150 \text{ Hz}$$
Problem 4  Sampling  (10 points)

A signal \( m(t) \) with a bandwidth of 40 Hz is sampled at 25% above the Nyquist rate. Suppose the resulting sample values are found to be

\[
\begin{align*}
  m(nT_s) &= -1, & \text{if } -6 \leq n \leq 2, & \text{and} \\
  m(nT_s) &= +1, & \text{if } 3 \leq n \leq 5, & \text{and} \\
  m(nT_s) &= 0, & \text{otherwise}.
\end{align*}
\]

What is the value of \( m(nT_s) \) at \( t = 0.0375 \) second?

Answer:

The sampling rate is twice 40 Hz plus a 25% guard band above the Nyquist rate \( R \).
\[
R = 2 \times 40 \times 1.25 = 100 \text{ Hz}.
\]

The Nyquist interval is 0.01 second, therefore at \( t = 0.0375 \) second we are at \( n = 3 \) so \( m(nT_s) = +1 \). \( \Leftarrow \)

Problem 5  Pulse Code Modulation  (20 points)

Explain how pulse code modulation (PCM) differs from the pulse modulation schemes you identified in Problem 2 above.

Answer:

Pulse code modulation assigns a code of multiple pulses to represent discrete values defined by the quantization of levels, where each level is assigned a range of values. Thus, PCM used a defined code of \( n \) bits to represent \( 2^n \) assigned values. However, for PAM, PWM and PPM a single bit represents a set of analog values (that are indicated by an amplitude, position or width of the pulse).