Amplitude Modulation – Early Radio
EE 442 – Spring Semester
Lecture 6

\[ f_{LO} \gg f_{audio\ baseband} = f_m \]

http://www.technologyuk.net/telecommunications/telecom_principles/amplitude_modulation.shtml
Modulation Options

We will study:
- AM
- FM
- PM
Baseband versus Carrier Communication

**Baseband communication** is the transmission of a message as generated is transmitted.

**Carrier communication** requires the modulation of the message onto a carrier signal to transmit it over a different frequency band. We use modulators to do the frequency translation.

(Note: “Pulse modulated” signals, such as PAM, PWM, PPM, PCM and DM are actually baseband digital signal coding (and not the result of frequency conversion).

**Use of Sinusoidal Carrier Signal**: Using a sine waveform there are three parameters which we can use to “modulate” a message onto the carrier – they are the amplitude, frequency and phase of the sinusoidal carrier.
Amplitude modulation (AM) is a modulation technique where the amplitude of a high-frequency sine wave (called a radio frequency) is varied in direct proportion to the modulating signal \( m(t) \). The modulating signal contains the intended message or information – sometimes consisting of audio data, as in AM radio broadcasting, or two-way radio communications.

The high-frequency sinusoidal waveform \( (i.e., \text{carrier}) \) is modulated by combining it with the modulating signal using a multiplier or mixer (mixing is a nonlinear operation because it generates new frequencies).

Agbo & Sadiku; Section 3.2, pp. 84 to 99
Amplitude Modulation in Pictures

**Frequency Domain**

- **Tone-modulated AM signal**
  - 100 kHz carrier modulated by a 5kHz audio tone
  
- **Voice-modulated AM signal**
  - 100 kHz carrier modulated by an audio signal (frequencies up to 6 kHz)

**Time Domain**

- **Carrier Signal**
- **5 kHz Audio tone**
- **Modulating Sine Wave Signal**
- **Amplitude Modulated Signal**
Example: Voice Signal – 300 Hz to 3400 Hz Baseband

Symbol $m(t)$ represents the source message signal.

Time Domain Display
Voice Band for Telephone Communication

Voice Channel
0 Hz – 4 kHz

Voice Bandwidth
300 Hz – 3.4 kHz

PSTN or POTS

Frequency Domain

AM Modulation -- Radio
Representative Voice Spectrum for Human Speech

For the telephone AT&T determined many years ago that speech could be easily recognized when the lowest frequencies and frequencies above 3.4 kilohertz were cutoff.

Waveform as received from a microphone converting acoustic energy into electrical energy.

Fast Fourier transform of the above speech waveform showing energy over frequency from 0 Hz to 12 kHz.
A crystal radio receiver, also called a crystal set or cat's whisker receiver, is a very simple radio receiver, popular in the early days of radio. It needs no other power source but that received solely from the power of radio waves received by a wire antenna. It gets its name from its most important component, known as a crystal detector, originally made from a piece of crystalline mineral such as galena. This component is now called a diode.
Foxhole Radio (used in World war I)

http://bizarrelabs.com/foxhole.htm
Modern Mechanix (December 1952)

**DICK TRACY WRIST RADIO**

... for kids from 6 to 60
Wear it like a watch
Use it as a radio!

Shockproof—Safe!

It really works. You've seen it in comic strips—now it's available for gift giving. Uses Radar Crystal Detector as developed by U.S. Air Forces. Receives regular AM radio broadcasts. Can be connected with wire for use as telephone system or as extra personal speaker for your home radio. Nothing to wear out or replace. Aerial and ground required for better reception.

$2.98 P.P.

Leotone Radio Corp., 65 Dey St., Dept. SM, New York, N.Y.
Crystal Radio Receiver from 1922

Diagram from 1922 showing the circuit of a crystal radio. This common circuit did not use a tuning capacitor, but used the capacitance of the antenna to form the tuned circuit with the coil.

Galena (lead sulfide) was probably the most common crystal used in “cat's whisker” detectors.
Amplitude Modulation (DSB with Carrier)

Amplitude Modulation: The amplitude of a carrier signal is varied linearly with a time-varying message signal.

Carrier signal: \( c(t) = A_C \cdot \cos(\omega_c t + \theta) \) \{Note: Keep \( \omega_c \) & \( \theta \) fixed.\}

Only amplitude \( A_C \) is allowed to vary in AM: \( \phi_{AM}(t) = [A_C + m(t)] \)

\[ \phi_{AM}(t) = \left[ A_C + m(t) \right] \cdot \cos(\omega_c t) = A_C \cdot \cos(\omega_c t) + m(t) \cdot \cos(\omega_c t) \]
Amplitude Modulation (DSB with Carrier)

http://hyperphysics.phy-astr.gsu.edu/hbase/Audio/bcast.html
Phasor View of Amplitude Modulation

Example shows tone modulation

a) Amplitude modulation phasor diagram

Modulated oscillation is a sum of these three vectors and is given by the red vector. In the case of amplitude modulation (AM), the modulated oscillation vector is always in phase with the carrier field while its length oscillates with the modulation frequency. The time dependence of its projection onto the real axis gives the signal strength as drawn to the right of the corresponding phasor diagram.

Brown vector $\Rightarrow$ Carrier signal
Red vector $\Rightarrow$ AM modulated signal

https://inspirehep.net/record/1093258/plots
Phasor View of Amplitude Modulation

$$\phi_{AM}(t) = \text{Re} \left[ e^{j\omega Ct} \left( 1 + \frac{e^{j\omega_m t}}{2} + \frac{e^{-j\omega_m t}}{2} \right) \right]$$

Tone signal $\sim \cos(\omega_m t)$

Carrier $\sim \cos(\omega_C t)$

Tone modulation

\(\omega_C - \omega_m\)  \(+\omega_C\)  \(\omega_C + \omega_m\)
Phasor Interpretation of AM DSB with Carrier (continued)

Tone modulation

\[ V_{us} = \text{Voltage of upper sideband phasor} \]
\[ V_{ls} = \text{Voltage of lower sideband phasor} \]
\[ V_c = \text{Voltage of the carrier} \]

\[ V_{\text{max}} = V_c + V_{us} + V_{ls} \]
\[ V_{\text{min}} = V_c - V_{us} - V_{ls} \]

\[ -V_{\text{max}} = -V_c - V_{us} - V_{ls} \]

https://www.slideshare.net/azizulhoque539/eeng-3810-chapter-4
Double-Sideband Amplitude Modulation Spectrum

\[ \phi_{AM}(t) = m(t) \cdot \cos(\omega_c t) + A_C \cos(\omega_C t) = [A_C + m(t)] \cdot \cos(\omega_C t) \]

The spectrum \( \Phi_{AM}(\omega) \) is found from the Fourier transform of \( \phi_{AM}(t) \)

\[ FT[\phi_{AM}(t)] = \frac{1}{2} M(\omega - \omega_C) + \frac{1}{2} M(\omega + \omega_C) + \pi A [\delta(\omega - \omega_C) + \delta(\omega + \omega_C)] \]

https://en.wikipedia.org/wiki/Amplitude_modulation

*Agbo & Sadiku; Section 3.2.1; pp. 89 to 91*
AM Modulation Index Basics – Definition

The amplitude modulation (AM) modulation index $\mu$ can be defined as a measure of the amplitude variation upon a carrier.

When expressed as a percentage it is the same as the depth of modulation. In other words it can be expressed as:

$$\text{Modulation Index } \mu = \frac{m_p}{A_C}$$

where $A_C$ is the carrier amplitude, and $m_p$ is the modulation amplitude (peak change in the RF amplitude relative to its un-modulated value).

Example: An AM modulation index of 0.5 means the signal increases by a factor of 0.5, and decreases to 0.5, centered around its unmodulated level. See drawings below.

$$\phi_{AM}(t) = [A_C + m(t)] \cdot \cos(\omega_C t)$$
AM Modulation Index Basics – Examples

50% Tone Modulation

100% Tone Modulation

150% Tone Modulation

Overmodulation or Envelope Distortion
AM Overmodulation → Envelope Distortion

https://www.physicsforums.com/threads/amplitude-modulation-vs-beats.890811/
Power Efficiency of Amplitude Modulation

Given the AM signal: \( \phi_{AM}(t) = A_c \cos(\omega_c t) + m(t)\cos(\omega_c t) \)

The power in the carrier is \( P_c = \frac{A_c^2}{2} \)

The power in the sidebands (modulated message) is

\[
P_s = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} m^2(t)\cos^2(\omega_c t)dt = \frac{1}{2} \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} m^2(t)[1 + \cos(2\omega_c t)]dt
\]

But \( \int_{-T/2}^{T/2} m^2(t)[\cos(2\omega_c t)]dt = 0 \), and

we are left with \( P_s = \frac{1}{2} \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} m^2(t)dt = \frac{1}{2} P_m \)

That is, \( P_s \) is one-half the total message power \( P_m \).

In AM the power in the message (useful power) is the power in the sidebands. Now we can finally define power efficiency \( \eta \).

Agbo & Sadiku; Section 3.2.2; pp. 91 to 92
Power Efficiency in Amplitude Modulation (continued)

The power efficiency of a modulated signal is the ratio of the power in the message part of the signal relative to the total power of the modulated signal.

\[
\text{Power efficiency } \eta = \frac{\text{message power}}{\text{total power}} = \frac{\text{sideband power}}{\text{total power}}
\]

In symbols, \( \eta = \frac{P_s}{P_c + P_s} = \frac{\frac{1}{2} P_m}{P_c + \frac{1}{2} P_m} \), and \( P_c = \frac{A_c^2}{2} \)

\[
\therefore \quad \eta = \frac{P_m}{A_c^2 + P_m}
\]
Power Efficiency in Amplitude Modulation (continued)

In general, the form of $P_m$ is complicated and not known precisely. However, we can study AM power efficiency $\eta$ using tone modulation.

For tone modulation $m(t) = A_m \cos(\omega_c t) = \mu A_C \cos(\omega_c t)$,

$$P_m = \frac{A_m^2}{2} = \frac{(\mu A_C)^2}{2} \quad \text{and} \quad \eta = \frac{A_m^2}{2A_C^2 + A_m^2} = \frac{\mu^2}{2 + \mu^2}$$

Examples:

<table>
<thead>
<tr>
<th>Modulation index $= \mu$</th>
<th>$\eta = \frac{\mu^2}{2 + \mu^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.0303 or 3.03 %</td>
</tr>
<tr>
<td>0.5</td>
<td>0.111 or 11.1 %</td>
</tr>
<tr>
<td>1.0</td>
<td>0.333 or 33.3 %</td>
</tr>
</tbody>
</table>

Conclusion: AM power efficiency is very low (highly undesirable).
Preview: Categories of Amplitude Modulation

Baseband spectrum (message spectrum)

Conventional AM (Double-SideBand With Carrier)

Special cases of AM:
- Double-Sideband-Suppressed Carrier (DSB-SC)
- Single-Sideband /Upper Sideband SSB/USB
- Single-Sideband /Lower Sideband SSB/LSB

Also Vestigial Sideband and Amplitude Companded SSB
Generation of Amplitude Modulated Signals

Agbo & Sadiku present two methods for AM generation:

1. **Nonlinear AM modulator**
   Almost any nonlinearity will work, but a very inexpensive but strongly nonlinear device is the diode. Transistors are also nonlinear and work well as modulators.

2. **Switching AM modulator**
   Switching is an easily attained function with diodes and transistors in electronic circuits.

There is also a third method:

3. **Electronic multipliers (such as Gilbert cells)**
Diode Operation Applied to AM Modulators & Demodulators

1. As **nonlinear circuit components** (primarily the “square law” part)

2. As **“on-off” switches** (they have to be driven hard to do this)
Using Nonlinearity For Modulation (*i.e.,* AM Generation)

The diode is the nonlinear component (it has an exponential characteristic). Using a Taylor’s series we can express the diode current $i_D$ as (only first two terms of Taylor’s series),

$$i_D(t) = b_1 v_D(t) + b_2 v_D^2(t); \quad v_D(t) \text{ is diode voltage.}$$

The voltage across resistor $R$ is given by

$$x(t) = i_D(t)R = b_1 Rv_D(t) + b_2 Rv_D^2(t) = a_1 v_D(t) + a_2 v_D^2(t)$$
Using Nonlinearity For Modulation (continued)

We now can evaluate voltage \( x(t) \)

\[
x(t) = a_1 \left[ m(t) + A_c \cos(\omega_c t) \right] + a_2 \left[ m(t) + A_c \cos(\omega_c t) \right]^2
\]

\[
x(t) = a_1 m(t) + a_2 m^2(t) + \frac{a_2 A_c^2}{2} \left[ 1 + \cos(2\omega_c t) \right]
\]

\[
+ a_1 A_c \left[ 1 + \frac{2a_2 m(t)}{a_1} \right] \cos(\omega_c t)
\]

Applying the bandpass filter about \( \omega_C \), the output voltage \( y(t) \) is

\[
y(t) = \phi_{AM}(t) = a_1 A_C \left[ 1 + \frac{2a_2 m(t)}{a_1} \right] \cos(\omega_C t) \quad \Leftarrow \quad \text{(Eq. 3.23)}
\]

Note: For \( \mu \) to be less than unity, we must demand \( \frac{2a_2 m(t)}{a_1} < 1 \).
Using Nonlinearity For Modulation (continued)

Comments:
1. Can use a general nonlinear element (not just a “square law” device)
2. The filter can be as simple as a LC resonator
3. This is a about the simplest of all modulators (it is unbalanced)
Using General Nonlinearity For Modulation

\[ v_{out} = v_{DC} + Gv_{in} + Av_{in}^2 + Bv_{in}^3 + \cdots \]

Taylor’s series

Input signals: \[ v_{in} = A_{RF} \cos(\omega_{RF}t) + B_{LO} \cos(\omega_{LO}t) \]

\( v_{out} \Rightarrow \omega_{RF} \text{ and } \omega_{LO} \)

\( v_{out}^2 \Rightarrow (\omega_{RF} + \omega_{LO}), (\omega_{LO} - \omega_{RF}), 2\omega_{RF} \text{ and } 2\omega_{LO} \)

\( v_{out}^3 \Rightarrow (2\omega_{RF} + \omega_{LO}), (2\omega_{RF} - \omega_{LO}), (2\omega_{LO} + \omega_{RF}), (2\omega_{LO} - \omega_{RF}), 3\omega_{RF} \text{ & } 3\omega_{LO} \)

Conclusion: Nonlinearity generates new frequencies.
Switching Amplitude Modulator – The Switch

\[ p(t) = \frac{1}{2} + \frac{2}{\pi} \left[ \cos(\omega_c t) - \frac{1}{3} \cos(3\omega_c t) + \frac{1}{5} \cos(5\omega_c t) - \cdots \right] \]

By driving a diode with sufficient AC voltage it acts like a switch:

**Forward bias**

By applying forward bias, the diode allows current to flow, effectively closing the switch.

**Reverse bias**

With reverse bias, no current flows, simulating an open switch.

\[ R_{on} = 0 \]
\[ R_{off} \text{ is infinite} \]
\[ \text{No Capacitance} \]
Switching Amplitude Modulator – Pulse Spectrum Generated

\[ p(t) = \frac{1}{2} + \frac{2}{\pi} \left[ \cos(\omega t) - \frac{1}{3} \cos(3\omega t) + \frac{1}{5} \cos(5\omega t) - \cdots \right] \]

\[ \tau = \text{Duty Cycle} \]

\[ \frac{\tau}{T} = \text{Duty Cycle} \]

Fourier series of the pulse train of period \( T \)

Fourier Series representation

Shown for a duty cycle of 1/4
Switching Modulator – Generating $m(t)\cos(\omega_C t)$

$p(t)$

Pulse train

$m(t)p(t)$
A “hopelessly unsophisticated” mixer.

— *Tom Lee (Stanford University)*

The unbalanced single-diode mixer has no isolation and no conversion gain.

Single-diode mixers have been used in many applications --

1. Detectors for radar in WW II
2. Early UHF Television tuners
3. Crystal radio detectors
4. mm-wave & sub-mm-wave receivers
AM Demodulation

Section 3.2.4 (pp. 95 to 99)

Coherent \textit{(i.e., synchronous) demodulation} (or detection) is a method to recover the message signal from the received modulated signal that requires a carrier at the receiver. This carrier signal must match in frequency and phase to the received signal.

But . . . Amplitude Modulation has the advantage of not requiring coherent detection methods. Non-coherent methods can be used which are much simpler to implement.

1. AM Envelope Detector

2. AM Rectifier Detector
AM Envelope Detector Circuit

Incoming AM modulated signal

Rectified AM modulated signal

Capacitor stores energy from the peaks of the rectified signal

Key idea: Capacitor captures the voltage peaks of rectified waveform

Envelope Detection requires the an RC network with time constant $\tau = RC$

Two conditions must be met for an envelope detector to work:

1. Narrowband [meaning $f_c >>$ bandwidth of $m(t)$]
2. $A_C + m(t) \geq 0$
Choosing the RC Time Constant in Envelope Detector

How the envelope is constructed

Time constant $\tau = RC$ too short.

Design criteria is $2\pi B < \frac{1}{RC} \ll 2\pi f_c$

Time constant $\tau = RC$ too long.
Practical Demodulation of an AM Signal

The user has a choice of changing the filter to meet their needs.
AM Rectifier Detection

\[ V_{rect}(t) = \left( (A_C + m(t)) \cos(\omega_C t) \right) \cdot p(t) \]

\[ = \left( (A_C + m(t)) \cos(\omega_C t) \right) \left\{ \frac{1}{2} + \frac{2}{\pi} \left[ (\cos(\omega_C t) - \frac{1}{3}(\cos(3\omega_C t) + \frac{1}{5}(\cos(5\omega_C t) - \ldots) \right] \right\} \]

\[ = \frac{1}{\pi} (A_C + m(t)) + \text{other terms.} \]

\[ = \text{dc term} + \text{baseband term} \]

Note: Multiplication with \( p(t) \) allows rectifier detection to act essentially as a synchronous detection without a carrier being generated at the receiver.
Double-Sideband Suppressed Carrier AM

Conventional AM transmits both the message and carrier signals. Hence, its power efficiency is low,

\[ \eta = \frac{P_m}{A_C^2 + P_m} < 100\% \]

If \( (A_C)^2 \) approaches zero, then \( \eta \) approaches 100%.

\[ \phi_{AM} (t) = [A_C + m(t)] \cdot \cos(\omega_C t) \]

For DSB-SC (double sideband -- suppressed carrier) we have

\[ \phi_{DSB-SC} (t) = m(t) \cdot \cos(\omega_C t) \quad \text{with a FT pair:} \quad m(t) \Leftrightarrow M(\omega) \]

\[ \text{FT} [m(t) \cos(\omega_C t)] = \Phi_{DSB-SC} (\omega) = \frac{1}{2} [M(\omega - \omega_C) + M(\omega + \omega_C)] \]
Double-Sideband Suppressed Carrier AM

Modulation:

\[ m(t) \rightarrow m(t) \cos(\omega_c t) \]

\[ \cos(\omega_c t) \]

Figure 3.11 in Agbo & Sadiku

DSB-SC Output

Note phase reversal
Double-Sideband Suppressed Carrier AM (continued)

\[ m(t) \iff M(\omega) \]

\[ M(\omega) \]

\[ -\Omega_m \quad 0 \quad +\Omega_m \]

\[ \Phi_{DSB-SC}(\omega) \]

\[ -\omega_c -\Omega_m \quad -\omega_c \quad -\omega_c +\Omega_m \quad 0 \quad +\omega_c -\Omega_m \quad +\omega_c \quad +\omega_c +\Omega_m \]

DSB-SC has USB and LSB spectra but not carrier impulses at \( \pm \omega_c \).
Double-Sideband Suppressed Carrier AM (continued)

Demodulation:

\[ x(t) = 2m(t) \cdot \cos^2(\omega_c t) = m(t) + m(t) \cdot \cos(2\omega_c t) \]

Use identity: \( \cos^2 \theta = \frac{1}{2} [1 + \cos(2\theta)] \)

The Fourier transform of \( x(t) \) is

\[ X(\omega) = M(\omega) + \frac{1}{2} [M(\omega - 2\omega_c) + M(\omega + 2\omega_c)] \]
Double-Sideband Suppressed Carrier AM (continued)

Let us examine the spectrum of the demodulated DSB-SC signal.

The message sidebands are shifted from being centered at ± \( \omega_c \) back to the about the origin (\( \omega = 0 \)) and ± 2\( \omega_c \). This is illustrated below.

Note that we now needed coherent (synchronous) detection!

We filter out the signals centered at and ± 2\( \omega_c \).
Double-Sideband Suppressed Carrier AM (continued)

Example 3.5 (on page 101):

We are given a carrier signal of $A_c \cos(\omega_c t)$ and a tone message signal of $m(t) = A_m \cos(\omega_m t)$

Therefore, the AM signal is

$$\phi_{DSB-SC}(t) = A_c A_m \cdot \cos(\omega_c t) \cdot \cos(\omega_m t) = \frac{A_c A_m}{2} \left[ \cos(\omega_c - \omega_m) t + \cos(\omega_c + \omega_m) t \right]$$

This comes from the identity: $\cos(\theta) \cdot \cos(\phi) = \frac{1}{2} \left[ \cos(\theta - \phi) + \cos(\theta + \phi) \right]$

Next we take the Fourier transform,

$$\Phi_{DSB-SC}(\omega) = \frac{\pi A_c A_m}{2} \left\{ \left[ \delta(\omega - \omega_c + \omega_m) + \delta(\omega + \omega_c - \omega_m) \right] + \left[ \delta(\omega - \omega_c - \omega_m) + \delta(\omega + \omega_c + \omega_m) \right] \right\}$$
Double-Sideband Suppressed Carrier AM (continued)

Tone modulation shown in example

\[ M(\omega) \]

\[ \Phi_{DSB-SC}(\omega) = \frac{\pi A_C A_m}{2} \]

\[ -\omega_c - \omega_m \quad -\omega_c \quad -\omega_c + \omega_m \quad 0 \quad +\omega_c - \omega_m \quad +\omega_c \quad +\omega_c + \omega_m \]
Analog Product Modulator

\[ V_{out} = kX \cdot Y \]

DSB-SC is used primarily today for point-to-point communications where a small number of receivers is involved.

One can buy commercial ICs that perform this function.

https://en.wikibooks.org/wiki/Electronics/Analog_multipliers
Non-Linear DSB-SC Modulator

\[ s_1(t) = a_1 m(t) + a_2 m^2(t) + \frac{a_2 A_C^2}{2} [1 + \cos(2\omega_C t)] + a_1 A_C \left(1 + \frac{2a_2 m(t)}{a_1}\right) \cos(\omega_C t) \]

\[ s_2(t) = -a_1 m(t) + a_2 m^2(t) + \frac{a_2 A_C^2}{2} [1 + \cos(2\omega_C t)] + a_1 A_C \left(1 - \frac{2a_2 m(t)}{a_1}\right) \cos(\omega_C t) \]

\[ s(t) = s_1(t) - s_2(t) = 2a_1 m(t) + 4a_2 A_C \cdot m(t) \cdot \cos(\omega_C t) \]

\[ \phi_{DSB-SC}(t) = 4a_2 A_C \cdot m(t) \cdot \cos(\omega_C t) \quad \Leftarrow \text{BPF selected this} \]

Refer to slide 29 for equations.
Non-Linear DSB-SC Modulator (continued)

\[ \phi_{DSB-SC}(t) = 4a_2 A_C \cdot m(t) \cdot \cos(\omega_c t) \]

Note that this expression contains no carrier signal. Why?

Answer: The modulator is a **balanced configuration** and this results in the carrier signal being cancelled (that assumes perfect balance of course).

Definition: A **balanced modulator** does not output either a carrier component or the message component. When both are missing we say it is **double balanced**.
Switching DSB-SC Modulators

Agbo & Sadiku present three switching modulators:

1. Series-bridge modulator
2. Shunt-bridge modulator, and
3. Ring modulator

We are only going to discuss the ring modulator because it is the most widely used and contains the fundamental principle of Operation of all of them. It is important you understand how it works.
Double-Balanced Diode Ring Modulator

The LO is driven hard enough to operate the diodes as on/off switches.

Bipolar square wave:

\[
p(t) = \frac{4}{\pi} \left[ \cos(\omega_c t) - \frac{1}{3} \cos(3 \cdot \omega_c t) + \frac{1}{5} \cos(5 \cdot \omega_c t) - \cdots \right]
\]

\[
m(t)
\]

\[
\nu_i = p(t) \cdot \cos(\omega_c t)
\]
Assume the diodes act as perfect switches (either “on” or “off”) and are controlled by the RF carrier signal (requires large amplitude).
Double-Balanced Diode Ring Modulator (continued)

Operation in the positive half-cycle of the carrier signal

Positive Half-Cycle:

\[ A_C \cos(\omega_c t) \]

\[ m(t) = 0 \]

Diodes \( D_3 \) & \( D_4 \) are Off

These currents cancel in the primary, thus, no output.
Double-Balanced Diode Ring Modulator (continued)

Operation in the negative half-cycle of the carrier signal

$A_C \cos(\omega_c t)$

$m(t) = 0$

Negative Half-Cycle:

Diodes $D_1$ & $D_2$ are Off

These currents cancel in the primary so no output.
Double-Balanced Diode Ring Modulator (continued)

Operation in the positive half-cycle of the carrier signal passes message signal $m(t)$ to output.

Diodes $D_1$ and $D_2$ are “on” and the secondary of $T_1$ is applied directly to $T_2$. 

$$A_C \cos(\omega_c t)$$
Double-Balanced Diode Ring Modulator (continued)

Operation in the negative half-cycle of the carrier signal inverts message signal $m(t)$ at the output.

Diodes $D_3$ and $D_4$ are “on” and the secondary of $T_1$ is applied directly to $T_2$. 
Double-Balanced Diode Ring Modulator (continued)

$$m(t)$$

**LO rectangular waveform**

**OUTPUT**

DSB-SC signal at primary of $$T_2$$

Green: $$D_1$$ and $$D_2$$ on
Blue: $$D_3$$ and $$D_4$$ on
Double-Balanced Diode Ring Modulator Waveforms

\[ m(t) \]

\[ A_C \cos(2\pi f_C t) \]

\[ p_{bipolar}(t) \cdot m(t) \]

\[ k \cdot m(t) \cos(\omega_C t) \]

After filtering

D₁ & D₂ are “on”

D₃ & D₄ are “on”
Double-Balanced Diode Ring Modulator

Requirements:

1. The carrier signal is higher in amplitude than the modulating signal $m(t)$.
2. The carrier signal must be of sufficient amplitude to fully switch the diodes between “on” and “off” states.
3. The carrier signal switches the diodes on and off at a rate higher than the highest frequency contained in $m(t)$.
4. The message signal $m(t)$ is chopped into segments, alternating between two amplitudes; $+m(t)$ and $-m(t)$.
Double-Balanced Diode Ring Modulator

The mathematics behind the Diode Ring Modulator:

\[ p_{bipolar}(t) = 2 \left( p(t) - \frac{1}{2} \right) = 2 p(t) - 1 \quad \Leftrightarrow \quad \text{This is bipolar square wave train.} \]

\[ p_{bipolar}(t) = 2 \left\{ \frac{1}{2} + \frac{2}{\pi} \left[ \cos(\omega_c t) - \frac{1}{3} \cos(3\omega_c t) + \frac{1}{5} \cos(5\omega_c t) - \cdots \right] \right\} \]

\[ p_{bipolar}(t) = \frac{4}{\pi} \left[ \cos(\omega_c t) - \frac{1}{3} \cos(3\omega_c t) + \frac{1}{5} \cos(5\omega_c t) - \cdots \right] \]

Therefore, the output is found by the product,

\[ x(t) = m(t) p_{bipolar}(t) = \frac{4}{\pi} \left[ m(t) \cdot \cos(\omega_c t) - \frac{m(t)}{3} \cos(3\omega_c t) + \frac{m(t)}{5} \cos(5\omega_c t) - \cdots \right] \]

Upon passing through the BPF,

\[ \phi_{DSB-SC}(t) = \frac{4}{\pi} \left[ m(t) \cdot \cos(\omega_c t) \right] \]
Double-Balanced Diode Ring Modulator/Mixer

It is inexpensive and easy to build a ring mixer.
Commercial Diode Ring Mixer (Mini-Circuits)

It is even easier to buy a ring mixer component.

1. These diagrams show a typical diode ring quad (top) or FET passive mixer (bottom); the inset photograph shows a surface-mount-packaged mixer from Mini-Circuits (Brooklyn, NY).
Mixers

Frequency mixing $\rightarrow$ frequency conversion $\rightarrow$ heterodyning

A mixer translates the modulation around one carrier frequency to another frequency. In a receiver, this is usually from a higher RF frequency to a lower IF frequency. In a transmitter, it’s the inverse.

We know that a LTI circuit can’t perform frequency translation. Mixers can be realized with either time-varying circuits or non-linear circuits.
**Digression: What is Heterodyning?**

**Heterodyning** is a signal processing technique invented in 1901 by Canadian inventor-engineer Reginald Fessenden that creates new frequencies by combining or mixing two frequencies using a nonlinear device.

Using an electronic circuit to combine an input radio frequency signal (RF) with another signal that is locally generated (LO) to produce new frequencies (IF): one being the **sum** of the two frequencies and the other being the **difference** of the two frequencies.

[http://www.yourdictionary.com/heterodyning#SgeI5zTvo7VV96o7.99](http://www.yourdictionary.com/heterodyning#SgeI5zTvo7VV96o7.99)

**Applications of heterodyning:**
1. Used in communications to generate new frequencies.
2. Move modulated signals from one frequency channel to another.
3. Used in the superheterodyne radio receivers able to select from multiple communication channels.
Mixers Perform Frequency Translation

Let $\phi_{RF}(t) = m(t) \cdot \cos(\omega_C t)$ and $\phi_{IF}(t) = m(t) \cdot \cos(\omega_{IF} t)$

The local oscillator (LO) is proportional to $\cos(\omega_{LO} t)$

The mixer (or multiplier) output $x(t)$ is given by

$$x(t) = 2m(t) \cdot \cos(\omega_C t) \cdot \cos(\omega_{LO} t)$$

A. Choosing $\omega_{LO} = \omega_C - \omega_{IF}$, we have

$$x(t) = m(t) \left[ \cos((\omega_C - \omega_{IF} - \omega_C)t) + \cos((\omega_C - \omega_{IF} + \omega_C)t) \right]$$

$$x(t) = m(t) \cdot \cos(\omega_{IF} t) + m(t) \cdot \cos((2\omega_C - \omega_{IF})t)$$

Note: Used even property of cosines [i.e., $\cos(-\theta) = \cos(\theta)$]

B. Choosing $\omega_{LO} = \omega_C + \omega_{IF}$, then we have

$$x(t) = m(t) \left[ \cos((\omega_C + \omega_{IF} - \omega_C)t) + \cos((\omega_C + \omega_{IF} + \omega_C)t) \right]$$

$$x(t) = m(t) \cdot \cos(\omega_{IF} t) + m(t) \cdot \cos((2\omega_C + \omega_{IF})t)$$
Frequency Conversion From $\omega_C$ to $\omega_{IF}$ With a Mixer

Multiplying a modulated signal by a sinusoidal moves the frequency band to sum and difference frequencies.

**Example**: We want to convert from frequency $\omega_C$ to frequency $\omega_{IF}$.

\[
\phi_{RF}(t) = m(t) \cdot \cos(\omega_C t) \quad x(t) \quad \phi_{IF}(t) = m(t) \cdot \cos(\omega_{IF} t)
\]

Input frequency $\omega_C$

\[2 \cdot \cos((\omega_C \pm \omega_{IF})t)\]

Output frequency $f_{IF}$

\[\omega = 2\pi f\]

**Note**: Super-heterodyning: $\omega_C + \omega_{IF}$; Sub-heterodyning: $\omega_C - \omega_{IF}$
Mixer Example (Page 110)

Example: Derive the relationship between \( \omega_{LO} \) and \( \omega_c \) so that centering the bandpass filter of the mixer is at \( \omega_{LO} - \omega_c \) and also ensure that \( \omega_{IF} \) is less than (i.e., below) \( \omega_c \).

Answer:

We know that \( \omega_{LO} = \omega_c \pm \omega_{IF} \) in general. We must meet two conditions:

\[
(1) \quad \omega_{IF} = \omega_{LO} - \omega_c \quad \text{and} \quad (2) \quad \omega_{IF} < \omega_c
\]

Start by assuming \( \omega_{LO} = \omega_c + \omega_{IF} \) that meets the first condition; then the second condition, \( \omega_{IF} < \omega_c \), implies that

\[
\omega_{LO} - \omega_c < \omega_c \quad \rightarrow \quad \omega_{LO} < 2\omega_c
\]
Superheterodyne Receiver is Widely Used

AM radio receiver:

A superheterodyne receiver, often called superhet, is a type of radio receiver using frequency mixing to convert a received signal to a fixed intermediate frequency (IF) which can be more conveniently processed than the original carrier frequency. It was invented by US engineer Edwin Armstrong in 1918 during World War I. Virtually all modern radio receivers use the superheterodyne principle.
Elenco AM/FM Dual-Radio Receiver Kit
10 kHz bandwidth from 540-1600 kHz for 106 possible bands

200 kHz bandwidth from 88.1-108.1 MHz for 100 possible bands
Another Mixer Example (Page 110)

For a frequency converter the carrier frequency of the output signal is 425 kHz and the carrier frequency of the AM input signal ranges from 500 kHz to 1500 kHz. Find the tuning ratio of the local oscillator

\[
\frac{\omega_{LO,\text{max}}}{\omega_{LO,\text{min}}},
\]

If the frequency of the local oscillator is given by (a) \(\omega_{IF} = \omega_{LO} - \omega_C\) and (b) \(\omega_{IF} = \omega_C + \omega_{LO}\).

**Answer:**

(a) \(\omega_{IF} = \omega_{LO} - \omega_C\) \(\rightarrow\) \(\omega_{LO} = \omega_C + \omega_{IF}\) \(\rightarrow\) superheterodyning

\[
\frac{\omega_{LO,\text{max}}}{\omega_{LO,\text{min}}} = \frac{\omega_{C,\text{max}} + \omega_{IF}}{\omega_{C,\text{min}} + \omega_{IF}} = \frac{1500 + 425}{500 + 425} = 2.081
\]

(b) \(\omega_{IF} = \omega_C + \omega_{LO}\) \(\rightarrow\) \(\omega_{LO} = \omega_C - \omega_{IF}\) \(\rightarrow\) sub-heterodyning

\[
\frac{\omega_{LO,\text{max}}}{\omega_{LO,\text{min}}} = \frac{\omega_{C,\text{max}} - \omega_{IF}}{\omega_{C,\text{min}} - \omega_{IF}} = \frac{1500 - 425}{500 - 425} = 14.33
\]
Quadrature Amplitude Modulation (QAM)

Fact: Both conventional AM and DSB-SC AM are wasteful of bandwidth.

One way to improve of bandwidth efficiency is with quadrature amplitude modulation (QAM). It involves two data streams: the I-channel and the Q-channel.

Bandwidth efficiency is improved by allowing two signals to share the same bandwidth of a channel. But this can only be done if the two modulated signals are orthogonal to each other. Let’s see how this can be accomplished.

A better name for this might be quadrature-carrier multiplexing.
Quadrature-Carrier Multiplexing

Quadrature-carrier multiplexing allows for transmitting two message signals on the same carrier frequency.

(1) Two quadrature carriers are multiplexed together,

(2) Signal $m_I(t)$ modulates the carrier $\cos(\omega_C t)$, and Signal $m_Q(t)$ modulates the carrier $\sin(\omega_C t)$.

(3) The two modulated signals are added together & transmitted over the channel as

$$\varphi_{QAM}(t) = m_I(t) \cdot \cos(\omega_C t) + m_Q(t) \cdot \sin(\omega_C t)$$
Quadrature Amplitude Modulation Features

1. QAM transmits two DSB-SC signals in the bandwidth of one DSB-SC signal.

2. Interference between the two modulated signals of the same frequency is prevented by using two carriers in phase quadrature. This is because they are orthogonal to each other.

3. The In-phase (I-phase) channel modulates the $\cos(\omega_c t)$ signal and the Quadrature-phase (Q-phase) channel modulates the $\sin(\omega_c t)$ signal.

4. The carriers used in the transmitter and receiver are synchronous with each other. In fact, they must be almost exactly in quadrature with each other, otherwise they experience cochannel interference.

5. Low-pass filters are used to extract the baseband signals $m_I(t)$ and $m_Q(t)$ in the receiver.
Quadrature Amplitude Modulation and Demodulation

\[ m_I(t) \]
\[ m_Q(t) \]

\[ m(t) = m_I(t) \cos(\omega_c t) + j m_Q(t) \sin(\omega_c t) \]

\[ z_I(t) = 2 \cdot m_I(t) \cos(\omega_c t) \]
\[ z_Q(t) = 2 \cdot m_Q(t) \sin(\omega_c t) \]

Transmitter

Receiver

Note: \( \cos(\omega_c t - 90^\circ) = \sin(\omega_c t) \)
Quadrature Amplitude Demodulation (QAM)

\[ \phi_{QAM}(t) = m_I(t) \cos(\omega_c t) + m_Q(t) \sin(\omega_c t) \]

\[ z_I(t) = 2 \cdot \cos(\omega_c t) \left[ \phi_{QAM}(t) \right] = 2 \cdot \cos(\omega_c t) \left[ m_I(t) \cos(\omega_c t) + m_Q(t) \sin(\omega_c t) \right] \]

\[ z_I(t) = 2m_I(t) \cdot \cos^2(\omega_c t) + 2m_Q(t) \cdot \cos(\omega_c t) \cdot \sin(\omega_c t) \]

\[ z_I(t) = m_I(t) + m_I(t) \cdot \cos(2\omega_c t) + m_Q(t) \sin(2\omega_c t) \]

We recover \( m_I(t) \) by passing \( z_I(t) \) through a LPF.

and

\[ z_Q(t) = 2 \cdot \sin(\omega_c t) \left[ \phi_{QAM}(t) \right] = 2 \cdot \sin(\omega_c t) \left[ m_I(t) \cos(\omega_c t) + m_Q(t) \sin(\omega_c t) \right] \]

\[ z_Q(t) = 2m_Q(t) \cdot \sin^2(\omega_c t) + 2m_I(t) \cdot \sin(\omega_c t) \cdot \sin(\omega_c t) \]

\[ z_Q(t) = m_Q(t) - m_Q(t) \cdot \cos(2\omega_c t) + m_I(t) \sin(2\omega_c t) \]

We recover \( m_Q(t) \) by passing \( z_Q(t) \) through a LPF.
Quadrature-Amplitude Modulation & Demodulation

\[ \phi_{QAM}(t) = m_I(t) \cos(\omega_c t) + m_Q(t) \sin(\omega_c t) \]
Quadrature-Amplitude Demodulation

Quadrature Downconverter

\[ m_I(t) = m(t) \cos(\omega_c t) \]
\[ m_Q(t) = m(t) \sin(\omega_c t) \]

\[ m_I(t) \]
\[ m_Q(t) \]

Re
Im

\[ m_I(t) \]
\[ m_Q(t) \]
**Review: Spectral Lines for Sine and Cosine Signals**

\[
\cos(2\pi f_c t) = \frac{e^{j2\pi f_c t}}{2} + \frac{e^{-j2\pi f_c t}}{2}
\]

\[
\sin(2\pi f_c t) = \frac{j e^{-j2\pi f_c t}}{2} - \frac{j e^{j2\pi f_c t}}{2}
\]
Quadrature-Amplitude Demodulation (continued)

This shows the orthogonality of the two modulated signals.

In-phase signal occupies the real axis-frequency plane

Quadrature signal occupies the imaginary axis-frequency plane
Quadrature-Amplitude Modulation & Demodulation

Now it becomes a digital communication system

\[ \phi_{QAM}(t) = m_I(t) \cos(\omega_c t) + m_Q(t) \sin(\omega_c t) \]
QAM: Phase Error in Synchronous Detection

The local carrier in a DSB-SC receiver and a QAM receiver is $2\cos(\omega_c t + \alpha)$, while the signal carrier at the input of each receiver is $\cos(\omega_c t)$. That means the signal carrier and local carrier in the receivers are phase shifted relative to each other. Derive expressions for the demodulated output signals for both receivers. Compare your results for DSB-SC and QAM.

Solution: (Example 3.9 on pp. 112-113)

For the DSB-SC receiver:

\[ x(t) = 2\cos(\omega_c t + \alpha) \cdot \phi_{DSB-SC}(t) = 2m(t) \cdot \cos(\omega_c t) \cdot \cos(\omega_c t + \alpha) \]

\[ x(t) = m(t) \cdot \cos(\alpha) + m(t) \cdot \cos(\omega_c t + \alpha) \]

Therefore, low-pass filtering gives

\[ y(t) = m(t) \cdot \cos(\alpha) \]
QAM: Phase Error in Synchronous Detection (continued)

For the QAM receiver:

\[ z_I(t) = 2 \cos(\omega_c t + \alpha) \cdot \phi_{QAM}(t) = 2 \cos(\omega_c t + \alpha) \left[ m_I(t) \cdot \cos(\omega_c t) + m_Q(t) \cdot \sin(\omega_c t) \right] \]

\[ z_I(t) = 2m_I(t) \cdot \cos(\omega_c t) \cdot \cos(\omega_c t + \alpha) + 2m_Q(t) \cdot \sin(\omega_c t) \cdot \cos(\omega_c t + \alpha) \]

\[ z_I(t) = m_I(t) \cdot \cos(\alpha) + m_I(t) \cdot \cos(2\omega_c t + \alpha) - m_Q(t) \cdot \sin(\alpha) + m_Q(t) \cdot \sin(2\omega_c t + \alpha) \]

and

\[ z_Q(t) = 2 \sin(\omega_c t + \alpha) \cdot \phi_{QAM}(t) = 2 \sin(\omega_c t + \alpha) \left[ m_I(t) \cdot \cos(\omega_c t) + m_Q(t) \cdot \sin(\omega_c t) \right] \]

\[ z_Q(t) = 2m_I(t) \cdot \cos(\omega_c t) \cdot \sin(\omega_c t + \alpha) + 2m_Q(t) \cdot \sin(\omega_c t) \cdot \sin(\omega_c t + \alpha) \]

\[ z_Q(t) = m_I(t) \cdot \sin(\alpha) + m_I(t) \cdot \sin(2\omega_c t + \alpha) + m_Q(t) \cdot \cos(\alpha) - m_Q(t) \cdot \cos(2\omega_c t + \alpha) \]

The low-pass filters suppress the terms centered at \(2\omega_c\).

Therefore,

\[ y_I(t) = m_I(t) \cdot \cos(\alpha) - m_Q(t) \cdot \sin(\alpha) \quad \text{Co-channel interference} \]

\[ y_Q(t) = m_Q(t) \cdot \cos(\alpha) + m_I(t) \cdot \sin(\alpha) \]
QAM: Frequency Error in Synchronous Detection

Compare the effect of a small frequency error in the local carrier for a DSB-SC receiver and a QAM receiver. The carrier at the transmitter is $\cos(\omega_C t)$ and the carrier at the receiver is $2\cos(\omega_C + \Delta \omega) t$.

Answer: *(Example 3.10 on pp. 113-114)*

\[
y_I(t) = m_I(t) \cos(\Delta \omega t) - m_Q(t) \sin(\Delta \omega t)
\]

\[
y_Q(t) = m_Q(t) \cos(\Delta \omega t) + m_I(t) \sin(\Delta \omega t)
\]

Note the similarity to the answer to Example 3.9. You should now be able to guess the answer to a question involving both phase error $\alpha$ and frequency error $\Delta \omega$. *(Practice Problem 3.10 on page 114)*
**Single Sideband (SSB) AM**

Why single sideband? DSB-SC is spectrally inefficient because it uses twice the bandwidth of the message. SSB addresses that issue.

The signal can be reconstructed from either the upper sideband (USB) or the lower sideband (LSB).

SSB transmits a bandpass filtered version of the modulated signal.
Single Sideband (SSB) AM

Multiplication of a USB signal by $\cos(\omega_c t)$ shifts the spectrum to left and right.
Phase-Shift Method to Generate SSB AM

\[
\phi_{SSB}(t) = m(t)\cos(\omega_c t) \mp m_h(t)\sin(\omega_c t)
\]

where minus sign applies to USB and plus sign applies to the LSB.

\(m_h(t)\) is \(m(t)\) phase delayed by \(-\pi/2\)
Phase-Shift Method to Generate SSB AM

• The phasing method uses two balanced mixers to eliminate the carrier.

• The phasing method for SSB generation uses a phase-shift to cancel one of the sidebands.

• The carrier oscillator is applied to the upper balanced modulator along with the modulating signal.

• The carrier and modulating signals are both shifted by 90 degrees and applied to another modulator.

• Phase-shifting causes one sideband to cancel when the two modulator outputs are summed together.
Phase-Shift Method for Receiving SSB Signals

Reference:

http://www.panoradio-sdr.de/ssb-demodulation/
Reference Note: Quadrature Phase-Shifts

For a + 90° (or $\pi/2$) phase shift:

\[
\begin{align*}
\sin(\omega t) & \Rightarrow -\cos(\omega t) \\
\cos(\omega t) & \Rightarrow \sin(\omega t)
\end{align*}
\]

For a - 90° (or $-\pi/2$) phase shift:

\[
\begin{align*}
\sin(\omega t) & \Rightarrow \cos(\omega t) \\
\cos(\omega t) & \Rightarrow -\sin(\omega t)
\end{align*}
\]

Hilbert transform

\[ H(\omega) = -j \cdot \text{sgn}(\omega) \]
Synchronous Demodulation of SSB AM


\[ x(t) = 2\phi_{SSB}(t) \cdot \cos(\omega_c t) = \left[ m(t) \cdot \cos(\omega_c t) + m_h(t) \cdot \sin(\omega_c t) \right] 2\cos(\omega_c t) \]

\[ x(t) = m(t) + \left[ m(t) \cdot \cos(2\omega_c t) + m_h(t) \cdot \sin(2\omega_c t) \right] \]

and upon low-pass filtering we have

\[ x(t) = m(t) \]
Hartley Image-Rejection Architecture

\[ \cos(\omega_c t) \sin(\omega_c t) \]

\[ \omega_{IF} \]

\[ \pm \omega_{IF} \]

\[ \omega_{LO} \]

\[ \pm \omega_{LO} \]

\[ 90^\circ \]

\[ \sum \]

http://www.microwavejournal.com/articles/3226-on-the-direct-conversion-receiver-a-tutorial
SSB Mixer and Image Rejection Mixer Comparison

**Single Sideband Mixer**

\[
\begin{align*}
& I \\
& \cos(\omega t) \quad \phi = 90^\circ \\
& Q \quad \sin(\omega t)
\end{align*}
\]

**Image Rejection Mixer**

\[
\begin{align*}
& I \\
& \cos(\omega t) \\
& Q \quad \phi = 90^\circ \\
& \sin(\omega t)
\end{align*}
\]

[https://commons.wikimedia.org/wiki/File:SSB_and_Image_Rejection_Mixer.svg](https://commons.wikimedia.org/wiki/File:SSB_and_Image_Rejection_Mixer.svg)
Pulse Amplitude Modulation (PAM) → Digital Signal

How is PAM in digital communication similar to AM in analog communication?
Questions

1. What is the point of creating a rectified output when using a diode for AM modulation?

It combines the carrier signal with the message signal $m(t)$. See slides 32, 33 & 34 for illustration of this.

2. More about how mixers work. (Three questions asked about mixers.)

Two principles are used in mixers to create new frequencies: (1) Nonlinearity the I-V characteristics of a nonlinear device do this, and (2) time-varying switching will create new frequencies. We provided examples of the use of both in slides 27 through 34.

3. Explain the idea of images in mixers again.

See the next four slides.
This converts the spectrum at the RF carrier frequency down to the spectrum centered at the IF frequency.

\[ \omega_{IF} = \omega_{LO} - \omega_{RF} \]

There is no signal in this part of spectrum.

Desired down-conversion
Image Signals in Mixers (2) – Now an Image Signal Appears

Now both the spectrum at the RF carrier frequency and the undesired image spectrum are down converted to the spectrum centered at the IF frequency.

The image spectrum is not wanted.

\[
\omega_{IF} = \omega_{LO} - \omega_{RF}
\]

and

\[
\omega_{IF} = \omega_{image} - \omega_{LO}
\]

Now both signals appear in the IF band.
Image Signals in Mixers (3)

This converts the spectrum at the RF carrier frequency down to the spectrum centered at the IF frequency.

Suppose the LO frequency is below the RF frequency.

Again, the desired down-conversion

\[ \omega_{IF} = \omega_{RF} - \omega_{LO} \]
Image Signals in Mixers (4) – With Image Signal

As before both the spectrum at the RF carrier frequency and the undesired image spectrum are down converted to the spectrum centered at the IF frequency.

The LO frequency is below the RF frequency.

\[ \omega_{\text{image}} \]

\[ \omega_{\text{image}} = \omega_{\text{RF}} - \omega_{\text{LO}} \]

and

\[ \omega_{\text{IF}} = \omega_{\text{LO}} - \omega_{\text{image}} \]

Again both signals appear in the IF band.
More Questions

4. Why does FM take so much more bandwidth than AM?

It is because we force FM (and PM) signals to have constant amplitude. We show this in detail when we cover Angle Modulation.

5. Is there a point where having too many mixers can impact your frequency negatively?

Issues with using multiple mixers:
   a. Adds complexity (such as many mixing products come into play)
   b. Most mixers are lossy and need power gain to continue to process signals (and mixers add noise of their own to signals)
   c. Cost (not only of mixers but for LO oscillators and amplifiers)
More Questions

6. In an antenna why does the length have to be $\lambda/4$?

It actually does not have to be $\lambda/4$, but making it longer does not have an advantage in maximizing its efficiency. Making it shorter does decrease the signal strength received. Also, we want the antenna to resonate at its operating frequency to increase the efficiency of the antenna.

7. Also, if an AM signal has a wavelength of hundreds of feet, how can its antenna be so small?

The coil of wire picks up the time-variation of the electromagnetic wave’s magnetic field and induces a current in the coil which becomes the signal at the input of the AM receiver.
More Questions

8. When you have no carrier signal, are you sending the message signal? If so, is there even any modulation of the amplitude or just the original signal?

In the absence of a carrier signal, then only the message signal can be sent at the frequency band of the message signal. We say the message signal “modulates” the carrier signal.

9. Does the local oscillator change its frequency depending upon the RF frequency you want to select?

I assume you are referring to the superheterodyne receiver. Yes, the local oscillator frequency is changed to select the RF frequency you want to select. We require that frequency difference between the RF and LO frequencies is a constant.
More Questions

10. How is a mixer similar/different from amplitude modulation?

The mathematics of the AM says we use a multiplier to multiply the carrier signal with the message signal. A mixer performs signal multiplication as required in amplitude modulation.

11. Why do we use sub-heterodyning? What are the common applications of sub-heterodyning?

When analyzing a system we focus upon the RF frequencies involved and possible local oscillator frequencies. The focus is upon frequency conversion. Considerations: Do we want to work with higher or lower frequencies? What frequency bands are to be avoided?

12. Which modulation is the most useful today? AM, FM or PM Why?

FM is the most widely used. It has better noise immunity.
More Questions

13. How do you build a mixer?

One example is slide 62 showing the diode ring mixer.

14. How would you handle having your oscillator being 1% off?

A 1% offset in frequency is very large. You might lock it to a reference frequency (such as a precision crystal oscillator). You might use a phase-lock loop to slave the local oscillator to the correct frequency. You might use a pilot signal broadcast with the modulated carrier. There are many other possible solutions.

15. From our homework assignments what would you consider the most important problems?

All of them. During the review session before the first midterm I will give a better answer.
More Questions

16. What happens to an over-modulated signal? Can the signal still be used?

Over-modulation leads to distortion in the message. It can still be used in voice communication if the voice over-modulation is not too severe. The point is that voice can still be understood even with moderate distortion.

17. Why do RLC resonator circuits have a -3 dB frequency corresponding to a bandwidth of \( B = \frac{1}{RC} \)?

\[
H(\omega) = \frac{i_R}{i(t)} = \frac{j\omega L}{R + j\omega L + (j\omega)^2 RCL}
\]

\[
\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + (LC)^2} \quad \text{&} \quad \omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + (LC)^2}
\]

Bandwidth \( B = \omega_2 - \omega_1 = \frac{1}{RC} \)
More Questions

18. How does \( x(t) = i_D(t)R \) come about? Why is it not KVL?

The diode is the nonlinear component (it has an exponential characteristic). Using a Taylor’s series we can express the diode current \( i_D(t) \) as (only first two terms of Taylor’s series),

\[
i_D(t) = b_1 v_D(t) + b_2 v_D^2(t) ; \quad v_D(t) \text{ is diode voltage.}
\]

The voltage across resistor \( R \) is given by

\[
x(t) = i_D(t)R = b_1 R v_D(t) + b_2 R v_D^2(t) = a_1 v_D(t) + a_2 v_D^2(t)
\]

“Square Law” behavior
More Questions

19. Can you explain using nonlinearity for modulation much like Problem 3 in Homework #3?

Given relationship

\( x(t) = i_D R = \beta \left( 4v_D + v_D^2 \right) \cdot R, \text{ but } R = 1 \Omega \)

Substituting for \( v_D = (\alpha + m(t) + A_c \cos(\omega_c t)) \),

\( x(t) = 4\beta \left[ (\alpha + m(t) + A_c \cos(\omega_c t)) \right] + \beta \left[ (\alpha + m(t) + A_c \cos(\omega_c t))^2 \right] \)

\( x(t) = 4\beta \left[ (\alpha + m(t) + A_c \cos(\omega_c t)) \right] + \beta \left[ (\alpha + m(t))^2 + 2A_c (\alpha + m(t) \cdot \cos(\omega_c t)) + A_c^2 \cos^2(\omega_c t) \right] \)

But we know \( A_c^2 \cos^2(\omega_c t) = \frac{A_c^2}{2} (1 + \cos(2\omega_c t)) \)

Continued next slide →
More Questions

Problem 3 in Homework #3 continued:

\[ x(t) = 4\beta \left[ (\alpha + m(t) + A_c \cos(\omega_c t)) \right] + \beta \left[ (\alpha + m(t))^2 + 2A_c (\alpha + m(t) \cdot \cos(\omega_c t)) + A_c^2 \cos^2(\omega_c t) \right] \]

\[ x(t) = \left( 4\beta(\alpha + m(t)) + \beta(\alpha + m(t))^2 + \frac{\beta A_c^2}{2} \right) + \left( 2\beta A_c (\alpha + m(t)) \cdot \cos(\omega_c t) \right) + \frac{\beta A_c^2}{2} \cos(2\omega_c t) \]

The band-pass filter (BPF) passes only terms of \( \cos(\omega_c t) \), thus \( y(t) \) is

\[ y(t) = 4\beta A_c \cdot \cos(\omega_c t) + 2\beta A_c (\alpha + m(t)) \cdot \cos(\omega_c t) \]

\[ y(t) = (4\beta A_c + 2\beta A_c \alpha) \cdot \cos(\omega_c t) + m(t) \cdot \cos(\omega_c t) \]

\[ y(t) = (K) \cdot \cos(\omega_c t) + m(t) \cdot \cos(\omega_c t) \]
More Questions

20. What is the primary form of noise production in AM systems?

The greatest noise problem in AM channels is interference, noise pickup, & fading in wireless transmission, all of which distort the amplitude of the transmitted signal.

21. QAM (Several asked about QAM so we need to cover it again)

I will review QAM again after questions are answered.
More Questions

22. Go over Problem 2 in Homework #2

Problem 2  Square Law Device  (20 points)

You are given a square-law component with an input to output relationship of

\[ y(t) = A + B(g(t))^2 \]

(a) To explore the behavior of this device we let the input signal \( g(t) \) be a sinusoidal tone, that is, \( g(t) = \cos(\omega t) \).

**Answer:** The square-law device generates a frequency that is the double of the single tone frequency \( f \).

To show this we make use of the trigonometric identity:

\[ y(t) = A + B[g(t)]^2 = A + B[\cos(\omega t)]^2 = A + \frac{B}{2}[1 + \cos(2\cdot\omega t)] \]

\[ y(t) = \left(A + \frac{B}{2}\right) + \frac{B}{2}\cos(2\cdot\omega t) \]

(b) What frequencies does the cubic term (that is, \( D[g(t)]^3 \)) generate when driven by \( g(t) = \cos(\omega t) \)?

\[ y(t) = A + Bg(t) + C(g(t))^2 + D(g(t))^3 + \text{other terms.} \]

**Answer:** The cubic term in the series generates a frequency that is triple of the frequency of \( g(t) \), that is, frequency \( 3f \). This comes from using the trigonometric identity of \( \cos^3(x) = \frac{1}{4}[3\cos(x) + \cos(3x)] \). Thus, the \( \cos^3(2\pi ft) \) term gives us both a \( \cos(2\pi ft) \) term (not so interesting) and a \( \cos(3\cdot2\pi ft) \) term (which is a new frequency being introduced).
More Questions

23. Why does milliwatts relate to dBm rather than just use milliwatts?

We express milliwatts (mW) in decibels by

\[
\text{Power in dBm} = 10 \cdot \log_{10} \left( \frac{P_{\text{mW}}}{1 \text{ mW}} \right) \text{ dBm} \quad \text{(Note: logarithm of a ratio)}
\]

We use this because logarithms add rather than multiply in calculations.

Example:

Suppose a signal of 3 dBm power drives an amplifier of gain = 13 dB. What is the output power of the amplifier.

Answer: Output power (in dB) = 3 dBm + 13 dB = 16 dBm, rather than 2 mW (= 3 dBm) multiplied by gain of 20 (= 13 dB) = 40 mW
More Questions

24. What is the one question you think we should have asked?

Answer: The one that is troubling you.
A Typical Superheterodyne Receiver
Generating \( m(t) \cdot \cos(\omega_C t) \) using Convolution

From Convolution Theorem: \( \sum(t) \cdot g(t) \Leftrightarrow C_n(\omega) \ast G(\omega) \)

Fold, Shift & Multiply