EE 442 Homework 5
(Due: March 4, 2019)

Question 1 AM Demodulation (30 points)

In this problem we want to demodulate an incoming double-sideband suppressed carrier AM signal of the form,

\[ \phi_{DSB}(t) = m(t) \cos(\omega_C t), \]

where \( m(t) \) is the message signal. The demodulator being used is is shown below:

\[ \phi_{DSB}(t) \]

\[ \times \]

\[ \text{LPF} \]

\[ y(t) \]

\[ c(t) \]

The local oscillator carrier signal is designated as \( c(t) \). You are given three different forms of \( c(t) \) for you to analyze.

(1) \( c_1(t) = e^{j\omega_C t} \)

(2) \( c_2(t) = e^{j(\omega_C t + \alpha)} \)

(3) \( c_3(t) = (\cos(\alpha)) \cdot e^{j(\omega_C t + \alpha)}, \)

where the parameter \( \alpha \) is a constant.

(a) For each of these three formats of \( c(t) \) find an expression for the output \( y(t) \).
Case (1)

Case (2)

Case (3)
(b) Which of the three cases give an undistorted replica of the message signal $m(t)$, except for a possible scaling in amplitude?

**Question 2  Frequency and Phase Error in AM Demodulation** (30 points)

In problem 2 above you investigated a mixer-based AM demodulator with different formats expressing the local oscillator carrier signals $c(t)$. In this problem you now have an input AM signal is of the form

$$\phi_{DSB}(t) = [A_C + m(t)] \cdot \cos(\omega_C t + \theta)$$

However, now the local oscillator signal to the mixer has both a frequency error $\Delta \omega$, and a phase error $\phi$. It is expressed as

$$c(t) = \cos\left((\omega_C + \Delta \omega) t + \phi\right)$$

**(a)** Derive an expression for the output of the mixer (that is, the IF output of the mixer going to the low-pass filter).
(b) If the frequency error $\Delta \omega = 0$ (i.e., no error), but the phase error $\phi$ is not zero, what is the expression for the mixer’s output?

(c) If the phase error $\phi = 0$ (i.e., no error), but the frequency error $\Delta \omega$ is not zero, then what is the expression for the mixer’s output?
**Question 3 Title**  (20 points)

You are given a dual-mixer AM demodulator as designed below:

The blocks immediately following the low-pass filters (LPF) are squaring components and the block following the summing node is a square root component (with only the positive root being selected).

Given an input AM signal of the form,

\[
\phi_{AM}(t) = [A_c + m(t)]\cos(\omega_c t + \theta),
\]

find the output \( y(t) \) of the demodulator.
Question 4  FM Radio Heterodyning and Band Images  (20 points)

The FCC has assigned FM broadcast radio a frequency band extending from 88 MHz to 108 MHz. Each FM radio station is allowed 200 kHz of bandwidth, providing for 100 stations across the FM spectrum. An FM radio superheterodyne receiver selects individual radio stations by tuning its local oscillator over a 20 MHz bandwidth such that it produces a 10.7 MHz IF signal with a 200 kHz bandwidth (it is a double sideband spectrum). The mixer converts the RF FM signal to a 10.7 MHz IF signal which is then processed as indicated in the block diagram below (filters are not shown in the block diagram). Remember that a mixer produces both sum and difference frequencies from the RF and LO (local oscillator) frequencies.

\[ f_{IF} = 10.7 \text{ MHz} \]

In this problem assume that an engineer has designed a variable oscillator capable of generating frequencies continuously from 98.7 MHz to 118.7 MHz.
(a) Show that this span of oscillator frequencies allows the FM receiver to select all FM stations within the 88 MHz to 108 MHz radio band. You might choose to use a frequency band diagram to illustrate this graphically.

(b) As the oscillator is swept from 98.7 MHz to 118.7 MHz, is it possible for image signals within the 88-108 MHz FM band to be present in the IF output of the mixer?

If yes, identify at least one image station signal frequency that could be picked up. If not, explain why it is not possible.