Problem 1  FM and PM Relationship  (8 points)

When we generate a “phase-modulated” signal modulated by message signal \( m_p(t) \); we simultaneously produce a frequency modulated signal. Of course, the frequency modulated signal corresponds to a different modulated waveform where we represent the message signal by \( m_f(t) \).

Write an equation stating the relationship between \( m_p(t) \) and \( m_f(t) \). [Hint: Equate the angles for both modulations.]

Problem 2  Mixer Used As a Phase Detector  (10 points)

You are presented with the FM signal generator as shown in the schematic circuit below. A voltage controlled oscillator (VCO) is controlled by a voltage from the output of the mixer (phase detector) added to a slowly varying input message signal \( m(t) \). The output of the VCO is an FM signal centered about the desired carrier frequency \( f_c \). The crystal oscillator operates at \( 1/N \) of the carrier frequency, where \( N \) is an integer. The reason for using a crystal oscillator is that it provides for a very stable frequency
(also, very stable crystal oscillators usually operate at lower frequencies than assigned carrier frequencies). Show how the mixer can be used as a phase detector outputting a voltage proportional to the phase difference between $f_{osc}$ and $f_c$. 

\[ f_{osc} = \frac{f_c}{N} \]
Problem 3  Frequency Swing of an FM Signal  (10 points)

In an FM system, a 7 kHz baseband signal modulates a 107.6 MHz carrier wave such that the frequency deviation $\Delta f$ is 50 kHz.

(a) Find the carrier frequency swing of the FM signal and the modulation index $\beta$. 
(b) Find the highest and the lowest frequencies attained by the FM signal.

Problem 4 Frequency Swing of an FM Signal (12 points)

(a) Determine the frequency deviation $\Delta f$ and the carrier swing for an FM signal with a carrier frequency of 100 MHz and whose upper frequency swing is 100.007 MHz when modulated by signal $m(t)$.

(b) Find the lowest frequency swing experienced by this FM signal.
Problem 5  FM Signal Parameters  (10 points)

An audio signal, with baseband of 200 Hz to 4 kHz, frequency modulates a carrier of frequency 50 MHz. The frequency deviation per volt is 10 kHz per volt and the maximum audio voltage it can transmit is 3 volts. Calculate both the frequency deviation $\Delta f$ and the bandwidth $BW$ of the FM signal.

Problem 6  FM Waveform  (20 points)

The figure below shows an FM carrier modulated by a single-tone sinusoidal wave. Calculate both the carrier frequency $f_c$ and the frequency of the tone frequency $f_m$. Express both frequencies in kilohertz (kHz).
Problem 7 Generating a WBFM Signal (30 points)

Design an Armstrong indirect FM modulator (generator) to generate an FM signal with carrier frequency $f_{c4} = 97.3000$ MHz and frequency deviation $\Delta f_4 = 10.240$ kHz. The block diagram of the indirect FM modulator is shown below.

A narrow-band FM generator at the left end of the block diagram generates a carrier frequency $f_{c1} = 20,000$ Hz and frequency deviation $\Delta f_1 = 5$ Hz. Only frequency doublers are available to build the two multipliers (of course, they can be cascaded together to obtain higher multiplication factors). Also, a local oscillator with adjustable frequency $f_{lo}$ that is tunable between 400 kHz and 500 kHz. However, once the LO $\Delta f$requency is determined, it is fixed in frequency. Find the multiplication values of $M_1$ and $M_2$ and find the local oscillator frequency $f_{lo}$.

These values are summarized in the table:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{c1}$</td>
<td>20,000 Hz</td>
<td>$f_{c4}$</td>
</tr>
<tr>
<td>$\Delta f_1$</td>
<td>5 Hz</td>
<td>$\Delta f_4$</td>
</tr>
<tr>
<td>$f_{lo}$</td>
<td>400 kHz &lt; $f_{lo}$ &lt; 500 kHz</td>
<td></td>
</tr>
</tbody>
</table>