Signal Transmission Through LTI Systems
EE 442   Lecture 3

The defining properties of any LTI system are \textit{linearity} and \textit{time invariance}.

From EE 400:

https://en.wikipedia.org/wiki/Linear_time-invariant_system
Steady-State Response in Linear Time Invariant Network

By steady-state we mean a sinusoidal excitation.

A sinusoidal signal of frequency $f$ at the input, $x(t)$, produces a sinusoidal signal of frequency $f$ at the output, $y(t)$. The output $y(t)$ is given by

$$Y(f) = H(f)X(f)$$

$H(f)$ will modify input $x(f)$ by a change in magnitude and phase. However, the frequency $f$ will be unchanged and the output will be causal.
Pulse Response in Linear Time Invariant Network

We are interested in the pulse response of a given LTI system with a bounded input – bounded output (BIBO).

\[ x(t) \xrightarrow{\text{LTI Network}} y(t) = x(t) \ast h(t) \]

where \( x(t) \) is the input, \( h(t) \) is impulse response of the network and \( y(t) \) is the output (Note: the symbol * denotes convolution).

\[ x(t) \Leftrightarrow X(f), \quad h(t) \Leftrightarrow H(f) \quad \text{and} \quad y(t) \Leftrightarrow Y(f) \]

where \( H(f) \) is the transfer function of the network.

We can write \( H(f) = |H(f)|e^{j\phi_h(f)} \)

and \( |Y(f)|e^{j\phi_y(f)} = |X(f)| \cdot |H(f)|e^{j(\phi_x(f)+j\phi_h(f))} \)
**Convolution Theorem (This should be review for you)**

The convolution theorem states that under suitable conditions the Fourier transform of a convolution operation is the product of Fourier transforms.
Example: Impulse Response of a LTI Network

This is special case of the transient response of a Linear Time Invariant (LTI) network.
Signal Distortion During Signal Transmission

In linear amplifiers and channel transmission we want the output waveform to be a replica of the input waveform. That means we would like to have **distortionless transmission**.

In other words: If $x(t)$ is the input signal, the output signal $y(t)$ is required to be

$$y(t) = K \cdot x(t - t_d)$$

($K$ is a constant)

$y(t)$ has its amplitude modified by factor $K$ and it is time delayed by time $t_d$.

In the frequency domain,

The Fourier transform is

$$Y(f) = K \cdot X(f) e^{-j2\pi f t_d}$$

by direct application of the convolution theorem.
Signal Distortion During Signal Transmission

For distortionless transmission the transfer function $H(f)$ is of the form,

$$H(f) = Ae^{-j2\pi ft_d}$$

$$|H(f)| = A \quad \text{and} \quad \phi_h(f) = -2\pi ft_d = -\omega t_d$$

Conclusion: Distortionless transmission requires a constant amplitude $|H(f)|$ over frequency and a linear phase response $\phi_h(f)$ passing through the origin at $f = 0$. 
**All-Pass System versus Distortionless System**

**All-Pass System:** Has a constant amplitude response, but doesn’t have a linear phase response.

A distortionless system is always an all-pass system, but the converse is not true.

**The transmission phase characteristic, if it doesn’t have a constant slope, causes distortion.**

In practice, most LTI systems only approximate a linear phase response passing through \( f = 0 \). Therefore,

\[
t_d(f) = -\frac{1}{2\pi} \frac{d\phi_n(f)}{df}
\]

either has a constant slope, otherwise, the time delay \( t_d \) is frequency dependent.

**For EE 442:** Phase distortion is important in digital communication systems because a nonlinear phase characteristic in a channel causes **pulse dispersion** (*i.e.*, results in spreading) and causes pulses to interfere with adjacent pulses (called **interference** or **Inter-Symbol Interference**).
Common Types of Transmission Lines

Two-wire line

Coaxial
- RG-58
- RG-8X
- RG-8U

Microstrip

Waveguide

PC board substrate

http://www.slideshare.net/simenli/ch1-2-49340691
Time Delay of Ideal Transmission Line

What is the time delay of a coaxial transmission line of length $L$?
The delay varies linearly with the length of the transmission line.

$$t_d = \frac{\theta \text{ (radians)}}{\omega \text{ (radians/sec)}}$$
and

$$t_d = \frac{L}{\text{velocity}}$$

$Z_L = Z_S = 50 \, \Omega$

2 cycles $\rightarrow$ $4\pi$ radians delay shown
Phase Delay in an Ideal Transmission Line

For an air-filled coaxial transmission line the time delay is roughly 1 nanosecond \((10^{-9} \text{ second})\) per foot of physical length \(L\). For a transmission line with a polyethylene dielectric the time delay is of the order of 1.5 nanoseconds per foot of length.

\[
\omega = 2\pi f
\]

\[
t_d = -\frac{\Delta \theta}{\Delta \omega}
\]

Length = \(L\)
We have an RC low-pass filter, find the transfer function $H(f)$, and sketch $|H(f)|$, the phase $\phi_n(f)$ and delay $t_d(f)$.

The transfer function $H(f)$ is given by $y(f)/g(f)$,

$$H(f) = \frac{\frac{1}{RC}}{\frac{1}{RC} + j(2\pi f)} \quad \text{general form: } \frac{a}{a + j(2\pi f)} \text{ where } a = \frac{1}{RC}$$
**RC Network LPF Example (continued)**

\[ H(f) = \frac{\left(\frac{1}{RC}\right)}{\left(\frac{1}{RC}\right) + j(2\pi f)} \quad \text{general form:} \quad \frac{a}{a + j(2\pi f)} \quad \text{where} \quad a = \frac{1}{RC} \]

Therefore, the magnitude \(|H(f)|\) and phase \(\theta_n(f)\) are given by

**Magnitude:**

\[ |H(f)| = \frac{\left(\frac{1}{RC}\right)}{\sqrt{\left(\frac{1}{RC}\right)^2 + (2\pi f)^2}}, \quad \text{and} \]

\[ \theta_n(f) = -\tan^{-1}\left(\frac{b}{a}\right) = -\tan^{-1}\left(\frac{2\pi f}{\left(\frac{1}{RC}\right)}\right) \quad \text{where} \quad b = 2\pi f \]

**Phase:**

\[ \theta_n(f) \cong -\left(\frac{2\pi f}{\left(\frac{1}{RC}\right)}\right) = -\frac{\omega}{a} \quad \text{for} \quad |2\pi f| \ll \left(\frac{1}{RC}\right) \quad \text{(low frequency)} \]
RC Network LPF Example (continued)

The time delay \( t_d = \) negative derivative of the phase with respect to frequency, therefore,

\[
 t_d(f) \equiv - \frac{d\theta_h(f)}{d\omega} = - \frac{d\theta_h(f)}{d(2\pi f)} = - \frac{1}{2\pi} \frac{d\theta_h(f)}{df}, \quad \text{[By definition]}
\]

which is the slope of the phase versus frequency plot.

For our RC low-pass filter example the time delay is

\[
 t_d(f) = - \frac{d\theta_h(f)}{d(2\pi f)} = - \frac{d}{d(2\pi f)} \left( -\tan^{-1} \left( \frac{2\pi f}{1/RC} \right) \right) = \frac{d}{d\omega} \left( -\tan^{-1} \left( \frac{\omega}{a} \right) \right)
\]

From a table of derivatives:

\[
 \frac{d}{dx} (\tan^{-1} x) = \frac{1}{1 + x^2}
\]
**RC Network LPF Example (continued)**

\[ t_d(f) \triangleq - \frac{d \theta_h(f)}{d(2\pi f)} = - \frac{d}{d(2\pi f)} \left( -\tan^{-1} \left( \frac{2\pi f}{1/RC} \right) \right) \]

\[
t_d(f) = -\frac{-1}{1 + \left( \frac{2\pi f}{1/RC} \right)^2} \cdot \frac{d}{d(2\pi f)} \left( \frac{2\pi f}{1/RC} \right) = \frac{\left( \frac{1}{RC} \right)^2}{\left( \frac{1}{RC} \right)^2 + (2\pi f)^2} \cdot \frac{1}{\left( \frac{1}{RC} \right)}
\]

\[
t_d(f) = \frac{\left( \frac{1}{RC} \right)}{\left( \frac{1}{RC} \right)^2 + (2\pi f)^2} = \frac{RC}{1 + (2\pi fRC)^2}
\]

Of course, for very low frequencies:

\[ t_d(f) \approx \frac{1}{a} = RC \quad \text{for} \quad |2\pi f| = |\omega| \ll a = \frac{1}{RC} = \frac{1}{\tau} \]
**RC Network LPF Example (continued)**

Amplitude response within 2% of peak value

Amplitude response is 0.707 of peak value

\[ a = \frac{1}{RC} = \frac{1}{\tau} \]

\[ \theta_h(\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right) \]

Slope = - \( t_d \)

(constant phase shift asymptote)

From Lathi & Ding, 4\textsuperscript{th} ed., 2008; Figure 3.28 (p. 129)
RC Network LPF Example (continued)

\[ t_d(f) = \frac{1}{a} \]

\[ \frac{1}{a} = RC = \tau \]

From Lathi & Ding, 4th ed., 2008; Figure 3.28 (p. 129)
An Ideal Filter has an Infinitely Sharp Cutoff

The signal $g(t)$ is transmitted without distortion, but has a time delay of $t_d$.

$$|H(f)| = \Pi\left(\frac{f}{2B}\right) \quad \text{and} \quad \phi_h(f) = -2\pi ft_d ;$$

$$\therefore H(f) = \Pi\left(\frac{f}{2B}\right)e^{-j2\pi ft_d} \quad \text{and}$$

$$h(t) = F^{-1}\left[\Pi\left(\frac{f}{2B}\right)e^{-j2\pi ft_d}\right] = 2B\text{sinc}\left(2\pi B(t - t_d)\right)$$

CAUSALITY VIOLATION

From Lathi & Ding, 4th ed., 2008; Figure 3.29 (p. 130)
An Ideal Filter versus a Practical Filter – II

Impulse response $h(t)$ is the response to impulse $\delta(t)$ applied at time $t = 0$.

The ideal filter on the previous slide is noncausal $\rightarrow$ unrealizable.

The practical approach to filter design is for $h(t) = 0$ for $t < 0$.

$$\hat{h}(t) = h(t) \cdot u(t)$$

A good approximation if $t_d$ is large. Many different practical “non-ideal” filters exist, for example:
A Butterworth Filter is a low-pass filter with amplitude response of

\[ |H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^{2n}}} \]

for \( n \)th order filter,

where \( \omega_c \) is the filter cutoff frequency and \( n \) is the filter order.

Dr. Adam Panagos
Analytical Methods for Multivariable and Discrete-Time Systems

https://www.youtube.com/watch?v=dmzikG1jZpU
Butterworth Filter (Maximally Flat Magnitude)

Pole-zero diagram

$s = \sigma + j\omega$

$n = 3$

Unit circle

$L_2$

$L_4$

$L_{n-1}$

$C_1$

$C_3$

"Cauer topology"  $C_n$

$|H(j\omega)|^2 = \frac{H^2(0)}{1 + \left(\frac{\omega}{\omega_C}\right)^{2n}}$

$sj = \sigma_j + j\omega$

$4^{th}$ order Butterworth Low-Pass Filter

Sallen–Key topology
Butterworth Filter Impulse Response

Fourth-order filter \((n = 4)\)

After Lathi and Ding, 4\(^{th}\) ed. 2009; Figure 3.33 (p. 133)
Comparing Butterworth, Chebyshev & Bessel Filters

**Phase Delay versus Group Delay**

Phase response of $H(f)$ is $\theta_h(f)$, therefore

Phase delay (at one frequency $f_0$) is $\tau_{ph} = -\frac{\theta_h(f_0)}{2\pi f_0}$, and

Group delay (over a frequency band) $\tau_{gr} = -\frac{d\theta_h(f)}{d(2\pi f)}$

**Example:**

Input: $g(t) = A(t) \cdot \cos(2\pi ft + \phi)$

Output: $y(t) = |H(f)| \cdot A(t - t_{grp}) \cdot \cos(2\pi f(t - t_d) + \phi)$
Group Delay (or Envelope Delay)

**Group delay** $\tau_{gr}$ is an important way to describe a filter's pass-band characteristics.

**Group delay** is:

1. A measure of a network’s phase distortion.
2. The transit time of a signal’s envelope through a device versus frequency.
3. The derivative of the device's phase characteristic with respect to frequency (*i.e.*, mathematical interpretation).

Consider the simple example of a square wave, which as you know is composed of a many frequency components. A square wave remains square only if its frequency components stay in proper phase alignment with one another. If we pass the square wave through a network and expect it to remain square, then we need to ensure that the network doesn't misalign the phases of each of these frequency components.
Another viewpoint on Group Delay in networks:

Phase is never constant over frequency and each frequency doesn’t take the same amount of time to travel through a device or over a channel. Group delay variation causes distortion of signal waveforms as they pass through a device or over a channel.

Group delay is another way to characterize deviations from an ideal, linear phase delay for all frequencies.
Amplitude and Phase Response for a Bandpass Filter

\[ |H(f)| \]

\[ \text{arg}\{H(f)\} \]


Actual phase

Ideal linear phase
Group Delay (continued)

Phase delay $\tau_{ph}$  $\tau_{gr}$  Group delay

Input

Output

https://www2.units.it/ramponi/teaching/DSP/materiale/Ch4%282%29.pdf
Channel Impairments (An Overview)

- **Linear distortion** caused by impulse response.
  \[ y(t) = h(t) \ast g(t) \quad \Leftrightarrow \quad Y(f) = H(f)G(f) \]
  \( H(f) \) attenuates and phase shifts the signal.

- **Nonlinear distortion** (such as “clipping”)
  \[ y(t) = \begin{cases} y_p & y(t) > y_p \\ y(t) & -y_p < y(t) < y_p \\ -y_p & y(t) < -y_p \end{cases} \]

- **Random Noise** (independent additive random signals)

- **Interference** from other transmissions or sources

- **Self interference & ISI** (from reflections or multipath)
  
  ISI is intersymbol interference
Pulse Distortion in a Sine-Squared Pulse

This is typically what pulse distortion looks like in channels.

(a) Amplitude-frequency distortion and phase-frequency distortion combined

(b) Amplitude-frequency distortion

(c) Phase-frequency distortion

The Advent of Radio – Some History

British Post Office engineers inspect Guglielmo Marconi's wireless telegraphy (radio) equipment, during a demonstration on Flat Holm island, 13 May 1897. This was the world's first demonstration of the transmission of radio signals over open sea, a distance of 3 miles.

https://www.allatsea.net/vhf-radio-distress-call/
Question:
Why is wireless communication so challenging?

https://www.semanticscholar.org/paper/Evolution-toward-5G-multi-tier-cellular-wireless-An-Hossain-Rasti/bdd9347c8a98be34f6c7fee29b59fb5a6f800c71
Wireless Channel: Atmospheric Radio Wave Propagation

Is this transmission channel analogous to a linear time-invariant network?

Radio Signal Intensity Decreases with Inverse Square Law

Isotropic radiator shown

This is what limits the range.

https://courses.lumenlearning.com/astronomy/chapter/the-behavior-of-light/
Types of Radio Wave Propagation in Atmosphere

- **Space-wave propagation**
  - Satellite

- **Sky-wave propagation**
  - Limited to 2 to 30 megahertz (MHz)
  - Long-distance coverage is due to ionosphere reflection (U-shape due to variation in index of refraction)
  - Used in international broadcasting, amateur radio, & military communication

- **Line of Sight (LOS) propagation**
  - Above 30 MHz (SHF band)
  - EM propagates in straight line (LOS)
  - FM radio & TV analog broadcasting

- **Ground-wave propagation**
  - Below 2 MHz (LF band)
  - EM wave follows the earth contour due to diffraction
  - Applies to AM radio broadcasting

From EE101A Lecture (D. B. Estreich – Spring 2015)
Wireless Channel Wave Propagation

Signal Propagation through Wireless Channels

- The mechanisms behind electromagnetic wave propagation are diverse, but can generally be attributed to reflection, diffraction, and scattering.

- Most cellular radio systems operate in urban areas where there is no direct line-of-sight path between the transmitter and the receiver, and where the presence of high rise buildings causes severe diffraction loss.

- Due to multiple reflections from various objects, the electromagnetic waves travel along different paths of varying lengths.

- The interaction between these waves causes multipath fading at a specific location, and the strengths of the waves decrease as the distance between the transmitter and receiver increases.

- Propagation models that predict the mean signal strength for an arbitrary transmitter-receiver (T-R) separation distance are useful in estimating the radio coverage area of a transmitter.

https://slideplayer.com/slide/12737878/
Primary Causes of Fading

1. Free space loss (scattering, molecular absorption, fog or rain, etc.)
2. Multipath (scattering from objects, reflection, diffraction and refraction)
3. Mobility (Doppler shifting, rapid environment changes from moving)
4. Interference (from other communication systems)
5. Noise pickup from channel (also in transmitters and receivers)

In addition: the channel is usually always changing with time.
Wireless Channel “Fading”

In a wireless communication environment, many copies of the signals get combined at the receiver – some add constructively and some add destructively. Generally, the quality of the combined signal at the receiver is degraded from the original signal. This process of signal deterioration by the combination of multiple propagation paths is called “Fading.”

Present Wireless Communication Challenges

Today’s biggest wireless challenges:

1. Scarcity of radio spectrum
2. Mutual interference among users
3. Power consumption in portable user equipment
4. Complexity of software to support user mobility
5. Cost of infrastructure

All of this impacts the introduction of 5G Cellular.
Questions?

What is going on in this diagram?