**EE 442 Homework 5 Solutions**  
(Due: March 4, 2020)

**Question 1  AM Demodulation** (40 points)

An AM signal has the form

\[ \phi_{AM}(t) = [1 + m(t)] \cdot \cos(\omega_C t), \quad \text{where} \quad |m(t)| < 1 \]

The baseband bandwidth of \( m(t) \) is \( B = 10 \, \text{kHz} \) and \( B \ll f_C \) (\( \omega_C = 2\pi f_C \)).

You are given the receiver as shown below:

![Diagram](image)

The local oscillator frequency is \( f_1 \) (where \( f_1 < f_C \)) and the input filter is a bandpass filter with bandwidth \( B_X \) and centered at frequency \( f_X \). The IF filter following the mixer has bandwidth \( B_{IF} \) and centered at a frequency of 455 kHz.

**(a)** For this receiver, specify frequency \( f_X \), bandwidth \( B_X \), frequency \( f_1 \) and bandwidth \( B_{IF} \).

**Solution:**

Obviously, the input bandpass filter’s center frequency must equal the AM signal’s carrier frequency,

\[ f_X = \frac{\omega_C}{2\pi} = f_C \quad \text{and} \quad B_X = 2 \times 10 \, \text{kHz} = 20 \, \text{kHz} \]
and the bandpass filter’s bandwidth is twice the baseband bandwidth because it is double sideband. Knowing that the IF filter’s center frequency is 455 kHz and that the filter must still pass the same bandwidth as the input filter, then

\[
f_{IF} = f_c - f_1 = 455 \text{ kHz}; \text{ thus, } f_1 = f - 455 \text{ kHz} \quad \Leftarrow
\]

and \( B_{IF} = 20 \text{ kHz} \quad \Leftarrow

Next, we are given three candidates for the detector for receiving \( m(t) \). They are designated A, B and C in the figure below.

(b) Which of the three detectors can produce an output at node \( \oplus \) of \( m(t) \)? If any of these three detector circuits can output the baseband signal \( m(t) \), provide a proof of this.

Answer: Only Circuit B can do this \( \Leftarrow \)

Solution:

Circuit A is only a linear filter so it can’t frequency translate the IF signal down to baseband. \( \Leftarrow \)
Circuit C is a circuit that squares the IF signal and then filters it.

$$\phi_{IF}^2(t) = [1+m(t)]\cos(\omega_{IF}t) \times [1+m(t)]\cos(\omega_{IF}t)$$

$$\phi_{IF}^2(t) = m(t) + \frac{1}{2}[1+m^2(t)] + \frac{1}{4}[1+m^2(t)]\cos(2\omega_{IF}t)$$

The $\cos(2\omega_{IF}t)$ term will be filtered out by the low-pass filter. That leaves two terms,

Filter's Output $= m(t) + \frac{1}{2}[1+m^2(t)]$

Although we have $m(t)$ in the output, the square of $m(t)$ is a problem because it can’t be separated out and greatly distorts the message signal. Thus, circuit C does not work. $\Leftarrow$

Circuit B does give the desired output $m(t)$ because it frequency translates down to baseband. $\Leftarrow$

Proof: To make Circuit B work we choose the local oscillator frequency $f_2 = 455$ kHz.

Combining the IP signal and LO signal in the mixer, we have

$$\phi_{MIXER}(t) = [1+m(t)]\cos(\omega_{IF}t) \times \cos(\omega_2t)$$

$$\phi_{MIXER}(t) = [1+m(t)]\cos(\omega_{IF}t) \times \cos(\omega_{IF}t)$$

$$\phi_{MIXER}(t) = \frac{1}{2}[1+m(t)] + \frac{1}{2}[1+m(t)]\cos(2\omega_{IF}t)$$

Upon filtering out the $2\omega_{IF}$ term, we get

$$\phi_{MIXER}(t) = \frac{1}{2}[1+m(t)]$$

Therefore, we recover $m(t)$. $\Leftarrow$
Question 2 Quadrature Dual Mixer AM Demodulator (30 points)

You are given a dual-mixer AM demodulator as designed below:

![Diagram of the dual-mixer AM demodulator]

The blocks immediately following the low-pass filters (LPF) are squaring components and the block following the summing node is a square root component (with only the positive root being selected).

Given an input AM signal of the form,

\[ \phi_{AM}(t) = [A_C + m(t)] \cos(\omega_C t + \theta), \]

find the output \( y(t) \) of the demodulator.

**Solution:**

Considering the upper branch,

\[ \phi_{upper}(t) = [A_C + m(t)] \cdot \cos(\omega_C t + \theta) \cdot \cos(\omega_C t) \]

\[ \phi_{upper}(t) = \frac{1}{2} [A_C + m(t)] \cdot (\cos(\theta) + \cos(2\omega_C t + \theta)) \]

The output at the upper branch LPF is

\[ \phi_{upper}(t) = \frac{1}{2} [A_C + m(t)] \cdot \cos(\theta) \]
Next, considering the lower branch,

\[ \phi_{lower}(t) = [A_c + m(t)] \cdot \cos(\omega_c t + \theta) \cdot \sin(\omega_c t) \]

\[ \phi_{lower}(t) = \frac{1}{2}[A_c + m(t)] \cdot (\sin(-\theta) + \cos(2\omega_c t + \theta)) \]

The output from the lower branch LPF is

\[ \phi_{lower}(t) = -\frac{1}{2}[A_c + m(t)] \cdot \sin(\theta) \]

Now squaring each output and summing them together gives,

\[ \phi_{upper}^2(t) + \phi_{lower}^2(t) = \left( \frac{1}{2} \right)^2 [A_c + m(t)]^2 \cdot [\cos^2(\theta) \cdot \sin^2(\theta)] \]

The output from the summing node is

\[ \phi_{upper}^2(t) + \phi_{lower}^2(t) = \left( \frac{1}{2} \right)^2 [A_c + m(t)]^2 \cdot [1] \]

Next, taking the square root of \( \phi_{upper}^2(t) + \phi_{lower}^2(t) \) gives

\[ \phi_{output}(t) = y(t) = \sqrt{\left( \frac{1}{2} \right)^2 [A_c + m(t)]^2} = \left[ \frac{A_c}{2} + \frac{m(t)}{2} \right] \]

**Question 3  FM Radio Heterodyning and Images**  (30 points)

The FCC has assigned FM broadcast radio a frequency band extending from 88 MHz to 108 MHz. Each FM radio station is allowed 200 kHz of bandwidth, providing for 100 stations across the FM spectrum. An FM radio superheterodyne receiver selects individual radio stations by tuning its local oscillator over a 20 MHz bandwidth such that it produces a 10.7 MHz IF signal with a 200 kHz bandwidth (it is a double sideband spectrum). The mixer converts the RF FM signal to a 10.7 MHz IF signal which is then processed as indicated in the block diagram below (filters are not shown in
the block diagram). Remember that a mixer produces both sum and difference frequencies from the RF and LO (local oscillator) frequencies.

\[ f_{IF} = 10.7 \text{ MHz} \]

In this problem assume that an engineer has designed a variable oscillator capable of generating frequencies continuously from 98.7 MHz to 118.7 MHz.

(a) Show that this span of oscillator frequencies allows the FM receiver to select all FM stations within the 88 MHz to 108 MHz radio band. You might choose to use a frequency band diagram to illustrate this graphically.

Answer: The difference frequency at the IP port of the mixer is set at 10.7 MHz. The diagram below shows the range of station selection as the LO frequency is swept over its band (98.7 MHz to 118.7 MHz). The IF frequency \( f_{IF} \) equals \( 10.7 \text{ MHz} = | f_{LO} - f_{RF} | \).

The sum frequency at the output port of the mixer is of no consequence because it is positioned around 200 MHz.
(b) As the oscillator is swept from 98.7 MHz to 118.7 MHz, is it possible for image signals within the 88-108 MHz FM band to be present in the IF output of the mixer?

If yes, identify at least one image station signal frequency that could be picked up. If not, explain why it is not possible. You may use a frequency band diagram to illustrate this, thereby avoiding the writing of a long essay.

Answer: No. There are image frequencies satisfying \(|f_{RF} - f_{LO}| = 10.7\) MHz. The lowest RF frequency \(f_{RF}\) when \(f_{LO} = 98.7\) MHz still yielding \(f_{IF} = 10.7\) MHz is \(f_{RF} = 109.4\) MHz. This is shown in the figure below where the image RF band goes from 109.4 MHz to 129.4 MHz – this is above the 88 MHz to 108 MHz commercial broadcast FM radio band. Therefore, it is not possible for two FM stations to be present at the IF port with this LO choice. Of course, other communications in the 109.4 MHz to 129.4 MHz band could show up, but they are not FM stations (and many of them are not FM modulated communications).

These two plots fully define the answer.