Problem 1 Wavelengths in Radio Applications (15 points)

Radio waves propagate in free space (and in our atmosphere) at the speed of electromagnetic waves (e.g., light waves) – an EM wave velocity of \( v = 2.99792 \times 10^8 \) meters per second. For this problem use \( v = 3.00 \times 10^8 \) meters per second (m/sec). An important wave parameter for electromagnetic waves is the wavelength \( \lambda \) which is inversely related to the wave frequency \( f \) (cycles per second in units of Hertz). The relationship is as you know velocity equals wavelength times frequency (\( v = \lambda \cdot f \)).

The reason wavelength \( \lambda \) is important is because the wavelength is approximately the spatial resolving dimension of radar and antenna sizes scale with wavelength (e.g., long wavelengths requires large antennas where the antenna will be of the order of the wavelength in size for best transmission and reception).

To get a feel for the size of the free space wavelength \( \lambda \) for various radio communication systems fill out the table below:

<table>
<thead>
<tr>
<th>Radio Application</th>
<th>Frequency Band</th>
<th>Wavelength Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>AM broadcast radio</td>
<td>535 kHz to 1605 kHz</td>
<td>560.7 meters to 186.9 meters</td>
</tr>
<tr>
<td>FM broadcast radio</td>
<td>88 MHz to 108 MHz</td>
<td>3.409 meters to 2.778 meters</td>
</tr>
<tr>
<td>VHF Civil Aviation Band (example)</td>
<td>108 MHz to 136 MHz</td>
<td>2.778 meters to 2.206 meters (example)</td>
</tr>
<tr>
<td>GSM Cellular (Uplink)</td>
<td>890 MHz to 915 MHz</td>
<td>0.3371 meter to 0.3279 meter</td>
</tr>
<tr>
<td>Wi-Fi 802.11b/g/n</td>
<td>2.400 GHz to 2.497 GHz</td>
<td>0.1250 meter to 0.1201 meter (or 12.50 cm to 12.01 cm)</td>
</tr>
<tr>
<td>K-band Radar Sensor</td>
<td>24.125 GHz (narrowband)</td>
<td>0.01244 meter (or 1.244 cm)</td>
</tr>
</tbody>
</table>
Problem 2 Frequency Requirement for a Cellular Phone  (15 points)

In problem 1 you found that radio wavelengths cover a very broad span of values. For example, in broadcast AM radio (which has been around since the 1920s), the wavelengths over it band are very large. The photo of the AM broadcast antenna is designed to be one-quarter of a wavelength ($\lambda/4$). For a radio station broadcasting at $f = 1240$ kHz the wavelength $\lambda = 241.9$ meters = 793.7 feet (because 1 meter = 3.2808 feet). Therefore, a quarter wavelength $\lambda/4 = 198.4$ feet high.

For this problem we want to estimate how high a frequency must be to have a handheld cellular telephone without a separate antenna extruding from the case of the cell phone. Again, let us assume that we can use a quarter wavelength antenna in the direction of the height of the cell phone case (such as using the side of the case itself). Using the dimension of the height of your cell phone and setting that dimension equal to one-quarter wavelength, what frequency $f$ meets this requirement? [Note: This would be the lowest frequency you would allow for operation.]

Solution:
My cell phone was approximately 5 inches in height. This is equal to 12.7 cm (because 2.54 cm equals 1 inch). Setting 12.7 cm equal to $\lambda/4$ gives $\lambda = 50.8$ cm for the wavelength.

$$f = \frac{v}{\lambda} = \frac{3 \times 10^{10} \text{ cm/sec}}{50.8 \text{ cm}} = 5.906 \times 10^8 \text{ (sec}^{-1}) = 590.6 \text{ MHz}$$

Problem 3 Voltage Gain & Power Gain  (16 points)

Electrical engineers often specify, or characterize, circuit blocks and/or networks in terms of voltage gain and power gain. Voltage and power gains can be expressed either numerically or in decibels (see Handout #1 for a discussion of decibels). In this problem you are presented with the amplifier circuit shown diagrammatically shown below with input and output resistances and voltages levels as labeled.
Assume the amplifier is impedance matched at output and input. Calculate:

(a) The voltage gain ratio.
\[
\frac{V_{out}}{V_{in}} = \frac{10 \text{ (volts)}}{0.5 \text{ (volt)}} = 20
\]

(b) The voltage gain expressed in decibels (dB).
\[
20 \cdot \log_{10} \left( \frac{V_{out}}{V_{in}} \right) = 20 \cdot \log_{10} (20) = 26 \text{ dB}
\]

(c) The power gain ratio.
\[
P_{in} = \left( \frac{0.5 \text{ V}}{75 \Omega} \right)^2 = 3.33 \times 10^{-3} \text{ W} \quad \text{and} \quad P_{out} = \left( \frac{10 \text{ V}}{75 \Omega} \right)^2 = 1.333 \text{ W}; \quad \frac{P_{out}}{P_{in}} = 400
\]

(d) The power gain ratio expressed in decibels (dB).
\[
10 \cdot \log_{10} \left( \frac{P_{out}}{P_{in}} \right) = 10 \cdot \log_{10} (400) = 26 \text{ dB}
\]

Added note for the curious student: Cable line amplifiers (CATV) in cable television distribution systems typically use 75 ohm coaxial cable (rather than 50 ohm cable) because a 77 ohm coaxial cable provides the lowest loss per length of line. The highest peak power carrying capability in a coaxial cable is a cable with a 30 ohm characteristic impedance. Then a 50 ohm coaxial cable is a compromise between the two characteristic impedances.

**Problem 4 Voltage Gain & Power Gain continued** (18 points)
We have another amplifier but it has an output resistance $R_{out} = 150$ ohms (no longer 75 ohms). This is shown in the diagram below. Again, assume input and output impedance matching.

\[ V_{in} = 0.4 \text{ volt} \quad \text{AMP} \quad V_{out} = 10 \text{ volts} \]

\[ R_{in} = 75 \Omega \quad R_{out} = 150 \Omega \]

Calculate:

(a) The voltage gain ratio.
\[
\frac{V_{out}}{V_{in}} = \frac{10 \text{ (volts)}}{0.4 \text{ (volt)}} = 25
\]

(b) The power gain ratio.
\[
P_{in} = \frac{(0.4 \text{ V})^2}{75 \Omega} = 2.133 \times 10^{-3} \text{ W} \quad \text{and} \quad P_{out} = \frac{(10 \text{ V})^2}{150 \Omega} = 0.6667 \text{ W}; \quad \frac{P_{out}}{P_{in}} = 312.5
\]

(c) The power gain expressed in decibels.
\[
10 \cdot \log_{10} \left( \frac{P_{out}}{P_{in}} \right) = 10 \cdot \log_{10} (312.5) = 24.95 \text{ dB} \approx 25 \text{ dB}
\]

(d) Write an expression for the power gain in decibels in terms of $V_{in}$, $V_{out}$ and the ratio of $R_{in}$ to $R_{out}$.
\[
10 \cdot \log_{10} \left( \frac{P_{out}}{P_{in}} \right) = 10 \cdot \log_{10} \left( \frac{\left( \frac{V_{out}}{V_{in}} \right)^2}{R_{out}} \right) = 10 \cdot \log_{10} \left( \frac{\left( \frac{V_{out}}{V_{in}} \right)^2 \times R_{in}}{R_{out}} \right) =
\]
\[
= 20 \cdot \log_{10} \left( \frac{V_{out}}{V_{in}} \right) + 10 \cdot \log_{10} \left( \frac{R_{in}}{R_{out}} \right)
\]
Problem 5 Voltage Gain & Power Gain continued (16 points)

We have four circuit components cascaded together as shown on the block diagram below.

(a) Express the gains and losses in decibels (use the second column in the table below):

<table>
<thead>
<tr>
<th>Numerical power ratio</th>
<th>Power ratio in dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1/P_{in} = 1/3$</td>
<td>-4.77 dB</td>
</tr>
<tr>
<td>$P_2/P_1 = 25$</td>
<td>+13.98 dB</td>
</tr>
<tr>
<td>$P_3/P_2 = 1/5$</td>
<td>-6.99 dB</td>
</tr>
<tr>
<td>$P_{out}/P_3 = 9$</td>
<td>+9.54 dB</td>
</tr>
</tbody>
</table>

(b) What is ratio of $(P_{out}/P_{in})$ (both numerically and in decibels)?

\[
\frac{P_{out}}{P_{in}} = \left(\frac{1}{3}\right)\left(25\right)\left(\frac{1}{5}\right)(9) = 3 \cdot 5 = 15
\]

\[
10 \cdot \log_{10} \left(\frac{P_{out}}{P_{in}}\right) = 10 \cdot \log_{10} (15) = 11.76 \text{ dB}
\]

or \(-4.77 \text{ dB} + 13.98 \text{ dB} -6.99 \text{ dB} +9.54 \text{ dB} = 11.76 \text{ dB}\)

(c) If $P_{in} = 30 \text{ mW}$, what is $P_{out}$ in watts (W)?

\[
P_{out} = 15 \times P_{in} = 15 \times 30 \text{ mW} = 450 \text{ mW} = 0.45 \text{ W}
\]

\[
5 \text{ dB} = 10 \cdot \log_{10} \left(\frac{P_{out}}{P_{in}}\right) \quad \Rightarrow \quad 10^{\frac{5}{10}} = 10^{\log_{10}(P_{out}/P_{in})}
\]

so \(3.162 = \frac{P_{out}}{500 \text{ mW}}\)

\[
P_{out} = 3.162 \times 500 \text{ mW} = 1.58 \text{ W}
\]
Problem 6 Communication Link: Gain & Loss (20 points)

We have four circuit components cascaded together as shown on the block diagram below. The links (Link 1-2 and Link 2-3) are long stretches of transmission paths between the networks. Link 1-2 has a loss of 30 dB and Link 2-3 has a loss of 20 dB. We don’t know the power gain $G_1$ of Network 1, but we are told that with an input power of $P_{in1} = 500$ mW fed into Network 1, the power flowing into Network 2 is $P_{in2} = 100$ mW.

You are asked to calculate the following:

(a) The output power (in milliwatts) from Network 1.

\[
P_{in2} = P_{out1} \times L_{\text{Loss of Link 1-2}} = P_{out1} \times (0.001) = 100 \text{ mW}
\]

because a -30 dB loss is numerically $1/1000 = 0.001$

Thus, $P_{out1} = 1000 \times 100 \text{ mW} = 100 \text{ W}$

(b) The power gain (in decibels) of Network 1 (power gain denoted by $G_1$).

\[
P_{out1} = P_{in1} \times G_1 = 500 \text{ mW} \times G_1 = 100 \text{ W}; \quad G_1 = \frac{100 \text{ W}}{0.5 \text{ W}} = 200
\]

\[
G_1 = 10 \cdot \log_{10} (200) = 23 \text{ dB}
\]

(c) The overall power gain, or power loss, of the entire chain (i.e., calculate $P_{out}/P_{in1}$). You may express it in decibels.

\[
G_{\text{overall}} = G_1 - Loss_{1-2} + G_2 - Loss_{2-3} + G_3 = 23 - 30 + 16 - 20 + 16 \text{ dB} = +5 \text{ dB}
\]

\[
10 \times \log_{10} (G_{\text{overall}}) = 5 \quad \Rightarrow \quad G_{\text{overall}} = 3.162
\]

Note: This example illustrates the advantage of using gain and loss in decibels.

(d) The output power $P_{out}$ in watts.

\[
P_{out} = G_{\text{overall}} \times P_{in1} = (3.162) \times 500 \text{ mW} = 1581 \text{ mW} = 1.581 \text{ W}
\]
**Extra Credit: Conservation of Energy in the Limit (25 points)**

In your electronic circuit courses you learned that the power dissipated by current $I$ through resistor $R$ is Power $P = I^2 R$ and that the energy stored on a capacitor $C$ when charged to voltage $V_0$ volts is equal to $\frac{1}{2}(CV_0^2)$. In the simple $RC$ circuit shown above, the capacitor is initially charged to a voltage $V = V_0$ volts. After charging the capacitor, a switch is then closed and the capacitor discharges through the resistor $R$.

(a) As you probably know, the discharge rate decreases exponentially as the voltage across the capacitor decreases. The decaying voltage across the resistor at time $t$ is $V(t) = V_0 \exp[-t/RC]$. Find the total energy dissipated in the resistor when the capacitor has fully discharged.

Answer: The energy stored in the capacitor is simply $\frac{1}{2}C(V_0)^2$. All of this energy is dissipated in the Resistor $R$ when the switch is closed. This occurs over time as the voltage across the capacitor decays exponentially with time constant $\tau = RC$.

Start with $I(t) = \frac{V(t)}{R} = \frac{V_0 \exp(-t/RC)}{R}$; Power $\enspace P = I^2(t)R = \frac{V_0^2 \exp(-2t/RC)}{R}$

Energy is given by the integral of power $= \frac{V_0^2}{R^2} \int_0^\infty \exp(-2t/RC) \, dt$

$= \frac{V_0^2}{R^2} \frac{1}{(-2/RC)} \bigg|_0^\infty = -\frac{1}{2}CV_0^2 \left[ \exp(-\infty) - \exp(0) \right] = \frac{1}{2}CV_0^2$

(b) Now for the interesting part of the question (also difficult because it is a “paradox”). The answer to the total energy dissipation [part (a) above] does not depend on the value of the resistor $R$. Assume the resistance goes to zero ohms. What happens to the energy when $R = 0$ (where there is no resistance to absorb the energy)? [Hint: Do not try to find a mathematical solution to this problem, rather qualitatively discuss its resolution.] Creative answers will be rewarded!

**Resolution:** Since $\tau = RC$, for $R = 0$, the time $\tau$ is zero. Resistance $R = 0$ would suggest the discharge would be instantaneous; but that implies an infinite current which is physically impossible. Standard circuit theory assumes lumped circuit elements of zero dimension or physical size. Removal of the lumped component assumption leads to distributed circuit analysis. Of course, real circuits consist of non-zero (finite) size...
and wires do have residual resistance (*i.e.*, there are no perfect conductors in electronic circuits).

**Viewpoint 1** – The wire around the capacitor has a physical size (*i.e.*, length) and therefore, has a loop inductance. That means the loop current supports a magnetic field which stores energy (remember electric and magnetic fields do contain energy and electromagnetic radiation carries energy away from a circuit). A time varying current requires a corresponding voltage around the loop. Also the creation of the magnetic field slows down the response so it not instantaneous.

**Viewpoint 2** – A finite length of wire has a small residual resistance, hence, the time constant $\tau$ is really not zero (although it may be very short). The resistance $R$ in the wire under the condition of very high currents experience Joule heating (*i.e.*, $I^2R$ heating) and the wire itself rapidly becomes very hot. With a high enough current the wire’s temperature can exceed the melting temperature of the wire. If the wire exceeds the melting temperature it can explosively splatter molten metal in all directions. This is an often observed event – it has lead to engineer’s establishing maximum current ratings for commonly all used wires based upon their diameter and material composition. When a wire melts it causes an “open-circuit” gap leading to a spark emitting a pulse of electromagnetic radiation. Such a spark radiates energy which carriers the remainder of the energy away from the capacitor.

If $R$ is truly zero ohms, then we have a situation which can’t occur in actual circuits. But there is value in thinking about such limiting cases because it can focus us on a deeper understanding of what happens in electrical circuits.