Problem 1 Multiplier Circuit (30 points)

In Problem Set #3, Problem 1, you analyzed an RLC resonant circuit which has the ability to serve as a bandpass filter. The resonant frequency of this circuit is

\[ f_{\text{resonance}} = \frac{1}{\sqrt{LC}} \]

An RLC circuit finds use in selecting communication bands by tuning to the desired frequency.

Consider the multiplier circuit shown below consisting of a common-emitter transistor stage loaded with an RLC network at the collector node of the transistor.

A sinusoidal signal generator drives the base-emitter junction such that when \( v_s(t) \) exceeds the forward-bias base-emitter junction turn-on voltage of 0.7 volt a collector current flows. When \( v_s(t) \) is less than 0.7 volt, negligible collector current flows. This is illustrated in the figure below.
Apparently short current pulses of \( i_c(t) \) are generated by this circuit when driven as indicated in the above figure. The frequency \( f \) of the signal source establishes the period \( T \) of the train of current pulses of width \( \tau \). Assume the period \( T \) to be equal to 5\( \tau \). Let us investigate how we can use this circuit as a multiplier.

(a) Begin by making a sketch of the spectral output of the current pulse train as shown in the above figure. Assume the \( i_c(t) \) pulses are approximately rectangular and \( T = 5\tau \). Let \( f_0 = 1/T \) be the fundamental frequency in the pulse train’s spectrum.

Remember the current pulses can be expressed using a Fourier series and be sure to show any null values in the spectrum of \( I_C(f) \). [ \( i_c(t) \leftrightarrow I_C(f) \) ]

Answer: \( f_0 = 1/T \) and discrete line spacing is \( (1/\tau) \) hertz.
(b) By using the RLC collector load network as a resonant filter, how can you make a frequency multiplier out of this circuit?

Answer: Tuning the RLC circuit resonance to one of the harmonic frequencies in the spectrum of $I_C(f)$. For example, if one wanted the third harmonic, namely $3f_0$, then the component values of the RLC network would be chosen so that the criterion of

$$2\pi (3f_0) = \frac{1}{\sqrt{LC}}$$

is met. The quality factor of the RLC network would have to be high enough to adequately attenuate the adjacent harmonic frequencies and that determines the range of resistance values allowable to have sufficiently high $Q$.

(c) If the period $T$ of the input signal is $T = 1$ microsecond ($1\times10^{-6}$ second) and we want to use this circuit to select the third harmonic of the fundamental frequency $f_0$, what must the value of the inductance $L$ be given a capacitor's value of $C = 1\times10^{-8}$ farad?

Answer:

The fundamental frequency $f_0$ from knowing the period $T$ is given by

$$f_0 = \frac{1}{T} = \frac{1}{10^{-6} \text{sec}} = 1\times10^6 \text{Hz} = 1 \text{MHz}$$

We also know that we want to select out $3f_0$ from the spectrum generated by the pulse train. The resonant frequency depends upon $L$ and $C$ by

$$(3 \times (2\pi) f_0)^2 = \frac{1}{LC} = 3.55 \times 10^{14} = \frac{1}{L \cdot 1 \times 10^{-8}}$$

$$L = 2.81 \times 10^{-7} \text{ H}$$

(d) How would you use the circuit in this problem to select out the fifth harmonic frequency (that is, $f = 5f_0$)?

Answer: You could make the pulse width $\tau$ smaller so that the fifth harmonic no longer is positioned at a null on the pulse train’s spectrum. You could do this by making the turn-on time of the BJT shorter. Of course, you would also have to tune the RLC network accordingly so as to select $f = 5f_0$.

Problem 2  DSB-SC Modulator  (30 points)
In this problem the challenge is to design a DSB-SC modulator to generate a modulated signal, \( m(t) \cdot \cos(\omega_c t) \), where \( m(t) \) is the modulating signal that is baseband limited to \( B \) Hz (see spectrum in part (b) below). The figure shows the DSB-SC (double-sideband – suppressed carrier) modulator you are given to work with. What is unusual about this modulator is that the local oscillator driving the modulator is \( \cos(3(\omega_c t)) \) rather than simply \( \cos(\omega_c t) \) as commonly used in most communication systems. Since the filter type is not specified, you may choose any kind of filter you want (e.g., low-pass, band-pass, or high-pass).

(a) What kind of filter is needed to give the output equal to \( m(t) \cdot \cos(\omega_c t) \)?

The signal at the mixer’s output is (from trigonometric table Handout 2):

\[
y_{\text{mixer}}(t) = m(t) \cdot \cos^3(\omega_c t) = m(t) \left[ \frac{3}{4} \cos(\omega_c t) + \frac{1}{4} \cos(3\omega_c t) \right]
\]

The desired output is the \((3/4) \cdot m(t) \cdot \cos(\omega_c t)\) term. Its spectrum is centered at \( \pm \omega_c \). The remaining term at frequencies of \( \pm 3 \omega_c \) is unwanted and can be filtered out. Hence, we need a bandpass filter centered at \( \pm \omega_c \).

(b) Determine the signal spectra at the mixer’s output and the filter’s output. Indicate the frequency bands occupied by these spectra.
(c) What is the minimum value of carrier frequency $\omega_c$ that can be used?

**Answer:** The minimum value for $\omega_c$ is $2\pi B$.

(d) Would this modulator as drawn in the above block diagram still work if the local oscillator generator were changed to $\cos^2(\omega_c t)$?

In this case we will have

$$y_{\text{mixer}}(t) = m(t) \cdot \cos^2(\omega_c t) = \frac{m(t)}{2} \left[ 1 + \cos(2\omega_c t) \right]$$

so it will not work with $n = 2$.

(e) Will this modulator work if the local oscillator generator is of the form $\cos^n(\omega_c t)$, where $n$ is an integer subject to the range of $n \geq 2$?

Looking at trigonometric identities for $\cos^n(\omega_c t)$ contains a term $\cos(\omega_c t)$ when $n$ is odd (i.e., 1, 3, 5, \ldots). This is not true when $n$ is even. Thus, $\cos^n(\omega_c t)$ works only when $n$ is odd.

**Problem 3  Ring Mixer** (20 points)
In class lecture we discussed the operation of the “Ring Diode Modulator” used to generate DSB-SC “amplitude modulated” signals. Suppose we take the same circuit (we reproduce it below) and ask if it can also be used for demodulation.

If the RF input is a DSB-SC AM signal, explain how would you use this ring diode mixer to deliver the baseband output \( m(t) \) from transformer \( T_2 \)?

The LO signal needs to be synchronized to the alternating polarities within the incoming RF input. Thus, when the input signal is in the high state, then the LO must be phased so the diodes will to deliver this high state of the signal to the baseband output. However, during the low state of the input signal, the LO must reverse the polarity of the signal as it is delivered to the baseband’s output. By alternating the switching polarities the carrier’s chopping of the the signal \( m(t) \) is removed. Therefore, the output is simply \( m(t) \).

**Problem 4 Baseband Signal Recovery**  (20 points)

A frequency-translated baseband signal \( m(t) \) (frequency shifted by \( f_c \)) is given by

\[
v(t) = m(t) \cdot \cos(2\pi f_c t)
\]

We can recover \( m(t) \) by multiplying \( v(t) \) by a local oscillator signal given by \( \cos(2\pi f_c t + \theta) \). The parameter \( \theta \) represents a phase shift. In this problem you are asked to investigate the effect of the offset in phase angle \( \theta \).

(a) The modulation product of \( v(t) \) and \( \cos(2\pi f_c t + \theta) \) is passed through a low-pass filter rejecting the double-frequency \((2f_c)\) term. After filtering, what is the signal output?

\[
v(t) \cdot \cos(2\pi f_c t + \theta) = m(t) \cdot \cos(2\pi f_c t) \cdot \cos(2\pi f_c t + \theta)
\]

\[
= m(t) \left[ \frac{1}{2} \cos(2\pi f_c t - 2\pi f_c t - \theta) + \frac{1}{2} \cos(2 \cdot 2\pi f_c t + \theta) \right]
\]

\[
= \frac{m(t)}{2} \left[ \cos(-\theta) + \cos(2 \cdot 2\pi f_c t + \theta) \right]
\]

The filter removes the \( \cos(2 \cdot 2\pi f_c t + \theta) \) term, therefore

\[
= \frac{m(t)}{2} \cos(-\theta)
\]
(b) Next, using the result from part (a) above, what is the filter’s output when $\theta$ is equal to $\pi/2$ radians?

\[
\frac{m(t)}{2} \cos(-\theta) = \frac{m(t)}{2} \cos(-\pi/2) = 0
\]

(c) How much phase shift $\theta$ can be tolerated for a decrease no greater than 10% of the magnitude at the filter’s output?

For a 10% decrease in response we require that $\cos(-\theta) = 0.9$, which corresponds to $\theta = 25.8$ degrees.