Problem 1 Creating two signals delayed by 90° (30 points)

We have seen that it is necessary to be able to generate signals in quadrature of each other. That is, it is valuable to generate a pair of signals that are exactly ±90° (or ±π/2) phase shifted from each other. So far in EE442 you have been introduced to both the low-pass RC filter and the high-pass RC filter. A student proposes that we use a combination of them as shown below.

Both filters share the same input $v_{in}(t)$ (could be modulating signal $m(t)$) and both branches use identical resistor $R$ and capacitor $C$ values. You probably immediately recognize the upper branch to be a low-pass filter and the lower branch to be a high-pass filter.

(a) Write an equation for the voltage transfer function for the HPF upper branch \( i.e., H_{HPF}(j\omega) \) and the LPF lower branch \( i.e., H_{LPF}(j\omega) \).
(b) Next, write expressions for the phase $\phi$ versus frequency $\omega$ for both the HPF upper branch $\{\phi_{HPF}\}$ and the LPF lower branch $\{\phi_{LPF}\}$.

(c) Plot the phase versus frequency of the two branches on the graph below. Use this plot to prove that both signals are in quadrature (i.e., phase difference is $90^\circ$ for all frequencies).

(d) What problems might arise if you were to build this circuit for implementation in a practical analog communication system (i.e., build it on a printed circuit board using actual components)?

Problem 2  Repeated phase shifts by $+90^\circ$ (20 points)

In class we discussed what happens to a sinusoidal signal when phase shifted by $-90^\circ$ degrees (or $-\pi/2$). Fill in the boxes showing how an input of $\cos(\omega t)$ as it is phase shifted by $+\pi/2$ four times in sequence.
Problem 3 “Squaring Loop” Demodulation of DSB-SC (25 points)

We have an incoming DSB-SC amplitude modulated signal of the form, \( m(t) \cdot \cos(\omega_c t) \). A “squaring loop” is shown in the block diagram.

\( g(t) = m(t) \cdot \cos(\omega_c t) \)

(a) If the input is \( g(t) = m(t) \cdot \cos(\omega_c t) \), then what is the output of the squarer block (that is, output at point “A” in the block diagram)?

(b) What is the output of the band-pass filter (that is, at point “B” in the block diagram)?

(c) What properties are required of the band-pass filter?
(d) What is the output $y(t)$?

(e) Why is a limiter used in the block diagram?

Problem 4 Two-Step SSB Generation (25 points)

Another method to generate SSB signals is the Two-Step SSB Generator. This approach can be used when the modulating function $m(t)$ has a two-sided frequency spectrum $M(f)$ with negligible energy around zero frequency (i.e., around DC) as shown in the spectral plot $M(f)$ below. Note the spectrum $M(f)$ has a gap centered at $\omega = 0$ (meaning DC).

![Spectral Plot](image)

The two-step SSB modulator (or generator) produces an USSB signal with carrier frequency $f_C = f_1 + f_2$, where the first oscillator is set at frequency $f_1$ and the second oscillator is at frequency $f_2$. It also requires the first band-pass filter (BPF) to have a lower cutoff frequency equal to $f_1$ and the lower cutoff frequency of the second BPF to be frequency $f_1 + f_2$. 
(a) Demonstrate the two-step SSB system’s operation by showing the spectra at points A, B, C and D along the component chain. [Note: If you want to use tone modulation to show this assume $m(t)$ to be at frequency $\omega_m$.]

(b) How would you modify the system to produce a lower SSB (i.e., LSSB) output at point D?