ES 442 Homework #6 Solutions
(Spring 2016 – Due March 30, 2016)
Print out homework and do work on the printed pages.


**Problem 1 FM versus PM Waveforms** (20 points)

Sketch the phase modulation (PM) and frequency modulation (FM) signals that are produced by the sawtooth waveform $m(t)$ shown below:
Problem 2  Bandwidth of a FM Signal (10 points)

A 10 MHz carrier signal is frequency modulated by a sinusoidal signal of unity amplitude and with a FM frequency deviation constant \(k_f = 10 \text{ Hz/V}\). Find the approximate bandwidth of the frequency modulated signal if the modulating frequency (single tone) is 10 kHz.

Answer:
Start with the expression: \(\varphi_{FM}(t) = A_c \cos \left(2\pi f_c t + \frac{k_f A_m}{f_m} \cos(2\pi f_m t)\right)\)

Modulation Index \(\beta = \frac{k_f A_m}{f_m} = \frac{(10 \text{ Hz/V})(1 \text{ V})}{10^4 \text{ Hz}} = 10^{-3}\)

So this is clearly a narrowband FM (NBFM) case. Since it is NBFM we can use the equation that the bandwidth is approximately twice the modulating time frequency \(f_m\).

NBFM bandwidth \(B_T = 2f_m = 20 \text{ kHz}\)

Problem 3  Bandwidth of a FM Signal (10 points)

A 100 MHz carrier signal is frequency modulated by a sinusoidal signal of 75 kHz, such that the frequency deviation is \(\Delta f = 50 \text{ kHz}\). Find the approximate bandwidth of the frequency modulated signal.

Answer:
We first check to see if this is narrowband FM or wideband FM. To do this we calculate the modulation index \(\beta\) to see if \(\beta\) is much less than one radian, or if it is greater.

Modulation Index \(\beta = \frac{\Delta f}{B} = \frac{50 \text{ kHz}}{75 \text{ kHz}} = 0.667\)

Therefore, \(\beta\) is not much less than one radian, so it is wideband FM (WBFM), and the bandwidth is given by

WBFM bandwidth \(B_T = 2B(\beta + 1) = 2(7.5 \times 10^4 \text{ Hz})(1.667) = 250 \text{ kHz}\)

Problem 4  (10 points)

You are given the equation below for a FM waveform,
From this expression, estimate the bandwidth of the FM signal.

Answer:
Starting with the formula, \( \varphi_{FM}(t) = A_c \cos[(2\pi f_c t) + \beta \cos(2\pi f_m t)] \),

we immediately see can equate terms where \( \beta = 35 \) and \( f_m = 50 \text{ Hz} \) (from \( 2\pi f_m = 100\pi \)). Now we find the frequency deviation \( \Delta f = \beta f_m = 35(50 \text{ Hz}) = 1750 \text{ Hz} \).

\[
\text{WBFM bandwidth } B_T = 2(\Delta f + B) = 2(\Delta f + f_m) = 2(1750 + 50) = 3600 \text{ Hz}
\]

Problem 5 Multiplexed Signals (20 points)

You have two signal sources, one of frequency of \( (f_c + \Delta f) \) and the other of \( (f_c - \Delta f) \), which can be combined into a output \( y(t) \) as controlled by a switch. The switch is being toggled at a modulating rate of frequency \( f_m \). Sketch the approximate waveform \( y(t) \) at the output of the circuit. Note: This is time multiplexing the signals.

\[
A \cdot \cos(2\pi(f_c + \Delta f)t)
\]

\[
A \cdot \cos(2\pi(f_c - \Delta f)t)
\]

Answer:
The switch alternates between the two frequencies so that the waveform will look something like the sketch below.
Problem 6  Digital Modulation (10 points)

Using the result you obtained in Problem 5, compare your result to the digital modulation technique known as frequency shift keying (FSK). Refer to Section 7.8.1 of Lathi and Ding, pages 423-425. What do you conclude in this comparison?

Answer: They are essentially the same or equivalent. It is only the interpretation that is different.

Problem 7  Frequency Modulation (20 points)

A frequency modulator is supplied with a carrier at frequency $f_c = 5$ MHz and an audio tone signal of $A_m = 1$ volt amplitude and frequency $f_m = 1$ kHz. It produces a frequency deviation $\Delta f$ of 10 kHz as the output of the frequency modulator.

(a) The FM waveform described above is passed through a frequency multiplier with a total multiplication factor of 18, that is, $f_{out} = 18 \cdot f_c$. After passing through the frequency multiplier what is the frequency deviation $\Delta f$ at the output?

Answer: The multiplier applies to both $f_c$ and to $\Delta f$, thus, the increase in the frequency deviation is

$$\Delta f \text{ after multiplication is } \Delta f = 18 \cdot \Delta f = 18 \cdot 10 \text{ kHz } = 180 \text{ kHz}$$

Assumption: The frequency multiplier acts on both $f_c$ and $\Delta f$. 
(b) The output of the multiplier in part (a) is next input to a mixer stage with the LO oscillator frequency set at $f_{LO} = 55$ MHz. Find the sum-frequency output $f_o$ of the mixer’s IF port and also find the magnitude of the frequency deviation $\Delta f$.

Answer: 
A mixer produces both sum and difference frequencies at the IF port. The multiplied carrier frequency $f_c$ input to the RF port is 90 MHz, hence, the sum frequency is 90 MHz plus 55 MHz = 145 MHz and the difference frequency is 90 MHz minus 55 MHz = 35 MHz. A mixer simply frequency translates a signal so the frequency deviation $\Delta f$ remains the same value of 180 kHz as you found in part (a). Therefore, $\Delta f = 180$ kHz.

(c) What is the modulation index $\beta$ at the output of the modulator.

Answer: 
The modulation index $\beta$ is still the same value because the mixer did not change the frequency ranges of the modulation.

$$\beta = \frac{\Delta f}{f_m} = \frac{180 \text{ kHz}}{18 \text{ kHz}} = 10$$

(d) Next, we change the audio input to the modulator to $A_m = 2$ volts and the modulation tone frequency $f_m$ is reduced to 500 Hz at the modulator’s output. What is the modulation index $\beta$ and the frequency deviation $\Delta f$ with these changes?

Answer: 
The amplitude of the modulating voltage $V_m$ is increased from 1 volt to 2 volts and the modulating tone frequency $f_m$ is reduced to 500 Hz.

We are asked to find the modulation index $\beta$ and the frequency deviation $\Delta f$

$$\Delta f = k_f A_m, \quad \text{and} \quad \beta = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m}$$

Now we see that $\Delta f$ is doubled from it prior value, namely, $\Delta f = 2k_f$ and

$$\beta = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m} = \frac{2k_f}{500 \text{ Hz}},$$

but 500 Hz is one-half of the prior $f_m$, so $\beta = 4k_f$. 