Problem 1  NA & Acceptance Angle  (15 points)

(a) Calculate the numerical aperture (NA) of a step-index optical fiber having a core index of refraction $n_1 = 1.480$, a cladding index of refraction of $n_2 = 1.460$, and a core diameter of 100 microns (100 $\mu$m). The fiber is intended to be operated at a wavelength $\lambda = 850$ nm.

(b) Calculate the acceptance angle $\theta_{acc}$ and how are numerical aperture and acceptance angle related? Assume air at the interface at the end of the fiber.

(c) Calculate the $V$-number for this optical fiber. Will this fiber allow higher order modes (i.e., above the lowest mode) to propagate?

Problem 2  NA and Acceptance Angle  (15 points)

A step-index optical fiber has a core index of refraction $n_1 = 1.425$. The acceptance angle for light entering the fiber from air ($n = 1.0003$) is found to 8.50 degrees.

(a) What is the numerical aperture of the optical fiber?
(b) What is the index of refraction $n_2$ of the cladding of this optical fiber?

(c) If the optical fiber is submerged in water, then what would be the numerical aperture and acceptance angle?

**Problem 3  Core Diameter and $n_2$  (20 points)**

A optical fiber manufacturer wants to make a silica-core, step-index optical fiber with a $V$-number = 75 and a numerical aperture $NA = 0.30$, to be used at a wavelength $\lambda = 820 \text{ nm}$. If the core index of refraction $n_1 = 1.458$, determine the value of (a) the core diameter (in micrometers) of the fiber and (b) the index of refraction $n_2$ of the cladding.
Problem 4  Optical Link Maximum Length  (20 points)

An optical power of 150 microwatts (150 $\mu$W) is launched into a fiber link. The optical fiber has an attenuation constant of 0.52 dB/km. We want to determine the maximum possible length of the optical link without signal amplification being allowed. The conditions pertaining to the optical link are that the receiver’s detector can detect a signal as weak as 2 microwatts (2 $\mu$W), and every 10 km an optical fiber joint (splicing two fiber sections together) must be included in the link, with each joint having a loss of 1.5 dB. How long (in km) can this fiber link be subject to these limitations?
Problem 5  Optical Polarization  (20 points)

A system of three polarizers (polarizers P1, P2 and P3 aligned in series) is setup as shown below.

\[
\begin{array}{c}
\text{P1} & \text{P2} & \text{P3} \\
I_0 & I_1 & I_2 & I_3 \\
\theta = 45^\circ & \theta = \phi^\circ \\
\end{array}
\]

The incident light intensity \(I_0\) is 1 candela (the unit of luminous intensity). The direction of each polarizer is indicated by the dashed line.

(a) The light incident upon P1 is unpolarized (or randomly polarized). What is the light intensity \(I_1\) coming out of polarizer P1 relative to the incoming incident intensity \(I_0\)?

(b) Polarizer P2 is tilted 45 degrees with respect to polarizer P1. What is the light intensity \(I_2\) coming out of polarizer P2 relative to the incident intensity \(I_0\)?

(c) Finally, polarizer P3 is rotated by angle \(\phi\) relative to the orientation of polarizer P2. What angle \(\phi\) is required for the intensity \(I_3\) to be 5% of intensity \(I_0\)?
Problem 6  Modal Dispersion  (10 points)

Derive the equation below (show your derivation) for the total modal dispersion $\delta T_{\text{mod}}$ in a multi-mode fiber, namely

$$\delta T_{\text{mod}} \approx \frac{L(NA)^2}{2n_1c}$$

where $L$ is the length of the fiber link, $NA$ is the numerical aperture of the fiber, $n_1$ is the index of refraction of the core of the fiber, and $c$ is the speed of light.

What are the assumptions you made in deriving this expression?