Problem 1  Fiber-Optic System  (15 points)

Consider an optical fiber system as shown in the drawing below.

The loss of the fiber link is 0.2 dB/km and its length is \( L = 80 \) km. The loss of the optical filter is -0.5 dB and the gain of the amplifier (block \( G_3 \)) is +15 dB. Suppose that the receiver must have a power level of at least \(-3\) dBm to operate without bit errors. **Calculate the minimum transmitter power** \( P_{in} \) **to operate this fiber-optic system. Express your answer for** \( P_{in} \) **in both “dBm” and in “milliwatts”**.

**Solution:**

Fiber link loss: \( F_1 \) (dB) = \(-0.20 \text{ dB/km} \times 80 \text{ km} = -16 \text{ dB} \)

Filter loss: \( F_2 = -0.5 \text{ dB} \)

Amplifier gain: \( G_3 = 15 \text{ dB} \)

The minimum power needed at the receiver is \( P_{out} = -3 \text{ dBm} \)

\[
P_{out} = P_{in} + F_1 + F_2 + G_3
\]

\[-3 \text{ dBm} = P_{in} - 16 \text{ dB} - 0 \text{ dB} + 15 \text{ dB} \]

\( P_{in} \) is the transmitter power which is \( P_{in} = -1.5 \text{ dBm} \) \( \iff \)

\[
P(mW) = 1 \text{ mW} \times 10^{(P(\text{dBm})/10)} = \text{mW} \times 10^{(-1.5/10)} = 0.7079 \text{ mW} \]
Problem 2 Einstein B Coefficient (20 points)

For an atomic system at $T = 300$ Kelvin, the two-level spontaneous lifetime associated with a $2 \rightarrow 1$ transition is $\tau_{21} = 2$ nanoseconds. The energy difference between the two levels is $2.4 \times 10^{-19}$ joule. **Calculate the Einstein $A_{21}$ and $B_{21}$ coefficients**, assuming the velocity of light in the medium to be $3 \times 10^8$ meters/second. Also, assume the degeneracies of both levels to be equal or unity. [Note: Planck’s constant is $6.626 \times 10^{-34}$ joule-second and Boltzmann’s constant is $1.381 \times 10^{-23}$ joule/Kelvin.] Finally, don’t forget the correct units for the Einstein $A_{21}$ and $B_{21}$ coefficients.

**Solution:**

We start with calculating the spontaneous lifetime (Lecture 7, slide 9)

$$A_{21} = \frac{1}{\tau_{21}} = \frac{1}{2 \times 10^{-9} \text{ [sec]}} = 5 \times 10^8 \text{ sec}^{-1} \text{ or } 500 \text{ MHz}$$

Also, the frequency is determined from

$$\Delta E = hf \quad \text{thus, } \quad f = \frac{2.4 \times 10^{-19} \text{ [J]}}{6.626 \times 10^{-34} \text{ [J-sec]}} = 3.622 \times 10^{14} \text{ Hz}$$

To calculate the $B_{21}$ coefficient we use (from Lecture 7, slide 11)

$$\frac{A_{21}}{B_{21}} = \frac{8\pi hf^3}{c^3} \quad \text{thus, } \quad B_{21} = \frac{A_{21}c^3}{8\pi hf^3}$$

Substituting values gives,

$$B_{21} = \frac{(5 \times 10^8)(3 \times 10^8)^3}{8\pi(1.054 \times 10^{-34})(3.62 \times 10^{14})^3} = 1.75 \times 10^{22} \text{ meter}^3 \text{ joule-sec}^{-2}$$

**NOTE:** Units of $\frac{\text{meter}^3}{\text{joule-sec}^{-2}}$ can also be written as $\frac{\text{meter}}{\text{kilogram}}$

Problem 3 Spontaneous Rate to Stimulated Rate (30 points)

In a two-level atomic system (at $T = 300$ Kelvin) the energy levels are separated by an energy of $1.26 \times 10^{-19}$ joule. The population density $N_1$ of the lower energy level is $10^{19}$ cm$^{-3}$. **Calculate the following parameters** (assume any degeneracies associated with the atomic levels are equal so you can ignore them because of cancellation):
(a) Calculate the wavelength $\lambda$ and frequency $f$ of the light emitted.

**Solution:**

$$\Delta E = hf \quad \text{(Note: } h = 6.626 \times 10^{-34} \text{ J-sec)}$$

$$f = \frac{\Delta E}{h} = \frac{1.26 \times 10^{-19} \text{ J}}{6.626 \times 10^{-34} \text{ J-sec}} = 1.90 \times 10^{14} \text{ Hz}$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8 \text{ m/sec}}{1.90 \times 10^{14} \text{ sec}^{-1}} = 1.578 \times 10^{-6} \text{ m} \quad \text{[or 1578 nm]}$$

(b) Calculate the ratio of spontaneous emission rate to stimulated emission rate.

Note: Boltzmann’s constant is $1.381 \times 10^{-23}$ joule/Kelvin.

**Solution:**

From Lecture 7, slide 11 we get the relationship,

$$\frac{R_{\text{spontaneous}}}{R_{\text{stimulated}}} = \frac{A_{21}}{B_{21} \rho_f} = \exp \left[ \frac{hf}{kT} \right] - 1 \quad (k = 1.381 \times 10^{-23} \text{ J/K})$$

$$\frac{R_{\text{spontaneous}}}{R_{\text{stimulated}}} = \exp \left[ \frac{1.26 \times 10^{-19} \text{ J}}{(1.381 \times 10^{-23} \text{ J/K})(300 \text{ K})} \right] - 1 = \exp[30.413] - 1$$

$$\frac{R_{\text{spontaneous}}}{R_{\text{stimulated}}} = 1.615 \times 10^{13} \quad \text{(Note: unitless ratio)}$$

The reciprocal of this is $6.192 \times 10^{-14}$

(c) Calculate the ratio of stimulated emission rate to the absorption rate. You can use the fact that $\frac{R_{\text{stimulated}}}{R_{\text{absorption}}} = \frac{N_2}{N_1}$ where the $R$'s are transition rates and $N_2$ and $N_1$ are the population density of energy levels 2 and 1, respectively. Use Boltzmann’s law to equate the ratio of $N_2$ to $N_1$.

**Solution:**

The Boltzmann relationship tells us that
\[ N_2 = N_1 \cdot \exp\left[-\frac{hf}{kT}\right] \]

\[ N_2 = 1 \times 10^{19} \cdot \exp\left[-\frac{1.26 \times 10^{-19} \text{ J}}{(1.381 \times 10^{-23} \text{ J/K})(300 \text{ K})}\right] \]

The ratio of the stimulated emission rate to the absorption rate is

\[ \frac{R_{\text{stimulated}}}{R_{\text{absorption}}} = \frac{N_2}{N_1} = \exp\left[-\frac{1.26 \times 10^{-19}}{(1.381 \times 10^{-23})(300)}\right] = \]

\[ \frac{R_{\text{stimulated}}}{R_{\text{absorption}}} = \frac{N_2}{N_1} = \exp[-30.413] = 6.193 \times 10^{-14} \]

(d) Calculate the population density of the upper level (namely, \( N_2 \)).

**Solution:**

From the Boltzmann relationship we have

\[ N_2 = N_1 \cdot \exp\left[-\frac{hf}{kT}\right] \]

\[ N_2 = 1 \times 10^{19} \cdot \exp\left[-\frac{1.26 \times 10^{-19} \text{ J}}{(1.381 \times 10^{-23} \text{ J/K})(300 \text{ K})}\right] \]

\[ N_2 = 1 \times 10^{19} \cdot \exp[-30.413] \text{ cm}^{-3} \]

\[ N_2 = 6.193 \times 10^5 \text{ cm}^{-3} \]

**Problem 4 Fabry-Perot Mirrored Resonator** (20 points)

A Fabry-Pérot cavity semiconductor laser has the following properties: Internal loss of 50 cm\(^{-1}\), mirror reflectivity of \( R_1 = R_2 = 0.30 \), and the distance between the partially reflecting mirrors is 500 micrometers (i.e., 500 μm). **Calculate the mode spacing \( \Delta f \) and the minimum cavity gain required for the onset of laser oscillation** (express in units of reciprocal centimeters). The index of refraction \( n \) of the cavity material = 3.5. [Hint: Remember that the gain must equal the sum of two losses, namely the internal loss \( \tilde{\alpha} \) and the loss from the mirrors \( \alpha_{\text{mir}} \).]

**Solution:**

We begin with the longitudinal mode spacings (using Lecture 7, slide 21),
\[ \Delta f = \frac{c}{2nL} = \frac{3 \times 10^8 \text{ m/sec}}{2(3.5)(500 \times 10^{-6} \text{ m})} \]
\[ \Delta f = 8.571 \times 10^{10} \text{ Hz or 85.71 GHz} \]

Next, we want the minimum gain for lasing (refer to Lecture 7, slides 30 and 31).

\[ \bar{g} = \bar{\alpha} + \frac{1}{2L} \log_e \left( \frac{1}{r_1 r_2} \right) = \alpha_{\text{internal}} + \alpha_{\text{mirrors}} \]

where the first term is the internal losses in the material and the second term is the loss from the mirrors not being perfect reflectors. Therefore, the internal loss is \( \alpha_{\text{internal}} \) is given as 50 cm\(^{-1}\).

Next, we use want to determine the internal loss of the laser cavity given that the material loss is 50 cm\(^{-1}\). In general, the relationship we start with allows us to determine the cavity loss assuming a round trip in the cavity is twice its length (\( L = 500 \mu\text{m}, \) but a round trip distance is twice 500 \( \mu\text{m} = 1 \text{ cm} \)).

**WARNING:** There is a difference between the Naperian attenuation coefficient [which decays as \( \exp(-\alpha x) \)] and the decadic attenuation coefficient [which decays as \( 10^{-\alpha x} \)]. They are related by a factor of 4.343 because \( 10 \log_{10}(e) = 4.343 \). So the Naperian coefficient is divided by the factor 4.343 to give the decadic coefficient.

\[
\frac{P_{\text{out}}}{P_{\text{in}}} = \exp[\alpha_{\text{internal}} \cdot L] = \frac{P_{\text{in}}}{P_{\text{in}}} \times \exp[-(50/\text{cm}) \cdot 1 \text{ cm}] \\
\frac{P_{\text{out}}}{P_{\text{in}}} = \exp[-50] \quad \text{You should know that} \quad 10 \cdot \log_{10} \left( \frac{P_{\text{out}}}{P_{\text{in}}} \right) = -50 \text{ dB} \\
\text{Thus, we have} \quad 10 \cdot \log_{10} \left[ \exp(-\bar{\alpha} \cdot (1 \text{ cm})) \right] = -\bar{\alpha} \text{ cm}^{-1} \times 1 \text{ cm} \times 10 \cdot \log_{10}(e) \\
\text{So,} \quad 10 \cdot \log_{10} \left[ \exp(-\bar{\alpha} \cdot (1 \text{ cm})) \right] = -\bar{\alpha} \times 4.343 = -50 \text{ dB} \\
\text{since} \quad 10 \cdot \log_{10}(e) = 4.343 \\
\therefore \quad \bar{\alpha} = \frac{50}{4.343} \text{ cm}^{-1} = 11.51 \text{ cm}^{-1} \quad \{\text{and not } 50 \text{ cm}^{-1} \text{ as many used}\} \\
\text{The distance between the mirrors is} \quad L = 0.05 \text{ cm}, \quad \text{and} \quad R_1 = R_2 = 0.3. \quad \text{The mirror loss is given by} \\
\alpha_{\text{mirrors}} = \frac{1}{2L} \log_e \left( \frac{1}{R_1 R_2} \right) = \frac{1}{2(0.05)} \log_e \left( \frac{1}{0.3 \times 0.3} \right) \quad [\text{cm}^{-1}] \\
\alpha_{\text{mirrors}} = 10 \times (2.4079) = 24.07 \text{ cm}^{-1} \\
\text{Now we calculate the threshold condition (gain = loss),} \]
\[ \tilde{\gamma} = \tilde{\alpha} + \alpha_{\text{mirrors}} = (11.51 + 24.07) = 35.58 \text{ cm}^{-1} \]

which is the gain threshold for laser operation.

**Problem 5  Bandgap in a Semiconductor**  (15 points)

Explain the difference between direct bandgap and indirect bandgap semiconductors. What does it means for semiconductor laser operation?

**ANSWER:**

In a direct band gap semiconductor, the top of the valence band and the bottom of the conduction band align over each other implying the same values of momentum (that is, aligned \( k \) values). In an indirect band gap semiconductor, the maximum energy of the valence band occurs at a much different value of crystal momentum at the minimum energy of the conduction band.

The minimal-energy state in the conduction band and the maximal-energy state in the valence band are each characterized by a certain crystal momentum (\( k \)-vector) in the Brillouin zone. If the \( k \)-vectors are different, the material has an "indirect gap". The band gap is called "direct" if the crystal momentum of electrons and holes are the same in both the conduction band and the valence band – thus an electron can directly emit a photon. In an "indirect" gap, a photon can only be emitted if there is a momentum transfer to the crystal lattice. In an indirect bandgap material, for an electron to fall from the conduction band to the valence band not only must the energy be given off by the emission of a photon, but the momentum difference between the two states must be carried off by a phonon (lattice vibration).

We desire direct bandgap semiconductors to make semiconductor lasers and LEDs.

https://en.wikipedia.org/wiki/Direct_and_indirect_band_gaps

https://sites.google.com/site/pengchenhomepage/wikipage/absorption_coef