Problem 1  Responsivity of InGaAs & Ge photodiodes  (16 points)

In the responsivity $R$ versus wavelength plot below both germanium and InGaAs photodiodes are represented. We want to operate at a wavelength of 1550 nm.

(a) what is the responsivity $R$ and the quantum efficiency $\eta$ for germanium (Ge) and InGaAs?
(b) At what wavelength does InGaAs have its maximum quantum efficiency?

Problem 2  Avalanche photodiode current multiplication  (14 points)

We have a silicon avalanche photodiode with a quantum efficiency of 60% at a wavelength of 900 nm. Suppose that 0.6 \( \mu W \) of optical input power produces a multiplied output current of 4.5 \( \mu A \).

(a) What is the expected primary output photodiode current if multiplication were not present?
(b) What is the multiplication factor $M$ for the output photocurrent?

**Problem 3 Thermal Noise**  (12 points)

We know that all resistors exhibit thermal noise (or Johnson noise).

(a) Compute the mean-square noise voltage per unit bandwidth for a 50 ohm resistor, a 1000 ohm resistor and a 1 megohm resistor. The temperature is assumed to be $T = 293$ K. Next, determine the root-mean-square noise voltage per square-root of the bandwidth. Let the bandwidth be one hertz ($= 1$ Hz) for each. Fill out the table below.

<table>
<thead>
<tr>
<th>Resistor value</th>
<th>Mean-square noise voltage per hertz</th>
<th>Root-mean-square noise voltage per root-hertz</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 ohms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1000 ohms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,000,000 ohms</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) If the bandwidth $\Delta f = 1$ MHz, what is the noise voltage of each of the three resistors?

<table>
<thead>
<tr>
<th>Resistor value</th>
<th>Root-mean-square noise voltage $v_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 ohms</td>
<td></td>
</tr>
<tr>
<td>1000 ohms</td>
<td></td>
</tr>
<tr>
<td>1,000,000 ohms</td>
<td></td>
</tr>
</tbody>
</table>
Problem 4  Noise factor of a cascaded amplifiers  (20 points)

In this problem you are asked to derive the equation for cascading two amplifiers (actually for any two cascaded two-ports). The first amplifier has gain $G_1$, noise factor $F_1$ and adds noise power $N_1$ to its output; the second amplifier has gain $G_2$, noise factor $F_2$ and adds noise power $N_2$ to its output. The arrangement is shown below:

\[
\begin{align*}
\text{Amp 1} & \quad \text{Gain} = G_1; \\
& \quad \text{F} = F_1; \\
& \quad \text{Added noise} = N_1
\end{align*}
\]

\[
\begin{align*}
\text{Amp 2} & \quad \text{Gain} = G_2; \\
& \quad \text{F} = F_2; \\
& \quad \text{Added noise} = N_2
\end{align*}
\]

Given the definition of noise factor for a single two-port (here an amplifier) is given by the expression,

\[
F_1 = 1 + \frac{N_1}{G_1 \times N_{in}}
\]

If amplifier 2 is considered by itself, its noise factor $F_2$ is given by

\[
F_2 = 1 + \frac{N_2}{G_2 \times N_{out,1}}
\]

Derive an expression for the noise factor $F_{12}$ for the cascaded combination of both amplifiers as illustrated in the above figure. Express your answer for $F_{12}$ in terms of both $F_1$ and $F_2$. Show your work in the derivation.
Problem 5  Another PIN Photodiode Problem  (38 points)

A silicon photodiode has a quantum efficient $\eta = 0.65\%$ at a wavelength $\lambda$ of 0.8 $\mu$m. The mean photocurrent when the photodiode is illuminated with $\lambda = 0.8$ $\mu$m light of optical power 5 $\mu$W is $I_p = 2.1$ $\mu$A. Now assume the bandwidth of the photodetector is 20 MHz.

(a) What is the root-mean-square noise current of the shot noise in the photodiode? (4 points)
(b) What is the SNR in decibels (dB) when there is no dark current and only the photocurrent is flowing? (4 points)

(c) The photodiode has an internal capacitance of 8 picofarads (pF). Determine the minimum load resistance $R_L$ presented to the photodiode corresponding to the requirement of a bandwidth of 20 MHz. (4 points)

(d) Calculate the root-mean-square thermal noise current of this resistor assuming an environmental temperature of 25 °C. (4 points)
(e) The photodiode drives an amplifier to create a receiver block. The amplifier has a noise figure of 4 dB and an input capacitance of 7 picofarads (pF). We need to maintain the 20 MHz bandwidth requirement, but this will require a change in the resistance at the photodiode to amplifier input node. What resistance is now needed to meet these conditions? (4 points)

(f) With a noise figure of 4 dB, calculate the incident optical power $P_o$ required for a $SNR$ of 50 dB. (18 points)