Problem 1 Energy per Bit Calculation (10 points)

You are presented with a polar NRZ signal with amplitude plus/minus A. (Note: Lecture 17, slide 15 defines polar NRZ coding.) Suppose the bit period is represented by \( T_b \). We introduced a normalized signal-to-noise ratio; written as \( E_b/N_0 \) where \( E_b \) is the energy per bit and \( N_0 \) is the noise power per Hertz. In an optical fiber the signal is an electromagnetic wave with electric and magnetic fields. Write an expression for the energy of a single bit, that is, for \( E_b \) using the information that the electric field amplitude is plus or minus A (A has units of volts/meter) and the bit period. Hint: an electromagnetic field has one-half its energy in the electric field and the other half of its energy in the magnetic field.

Problem 2 Random Bit Streams (10 points)

As discussed in class on Thursday (November 14th) it is very difficult to generate a truly random number. For this exercise each of you are being asked to generate a sixteen bit binary sequence. I will take all of your inputs upon grading this homework and
analyze the entire set for randomness. You can just think up a binary sequence (if you can think randomly, without any bias) or use some other method you select.

(a) Write out your 16-bit random binary sequence (one’s and zero’s):

(b) Describe the method you used to generate the binary sequence.

Problem 3 Line Code Rules (10 points)

Given the unknown line code (X) shown below for the data sequence shown at the top of the figure.

(a) generate a written description of the rules for generating this line code.

(b) List the advantages and disadvantages of line code X.
Problem 4  Commonly Used Digital Coding  10 points)

Explain the advantages and the disadvantages of using the Manchester line code.

Problem 5  Non-concentric Misaligned Fibers  (30 points)

Consider the case where two optical fibers are being spliced together. Assume that both fiber ends are perfectly cleaved and there is no air space between the two ends of the fibers. However, they are not concentric, that is, misaligned. Both fibers have core radii $R = 25$ micrometers, and the centers of the cores are offset by a distance $d = 10$ micrometers. This is illustrated in the cross-sectional diagram below which defines the symbols used. Assume refractive index of both cores to be $n = 1.50$. 

![Cross-sectional diagram of non-concentric fibers](image-url)
The shaded area is the overlapping cross-sectional area determining the fraction of optical power transferred from one fiber to the mating fiber. Obviously, the optical power is reduced by the misalignment and we want to know by how much it is reduced as a function of distance $d$.

**a)** Derive an expression for the area of the shaded (overlapping) area. This derivation is a bit challenging, so if you get stuck you might want to refer to [https://diego.assencio.com/?index=8d6ca3d82151bad815f78addf9b5c1c6](https://diego.assencio.com/?index=8d6ca3d82151bad815f78addf9b5c1c6) But at least attempt to derive it before you consult the above URL.

**b)** What fraction of the optical power is transmitted using the values for the fiber core radius ($R = 25 \, \mu m$) and misalignment separation is $d = 10 \, \mu m$. State any assumptions you made. Remember that when $d = 0$, the area $A_0 = \pi R^2 \, (\mu m)^2$. .
Problem 6  Two Aligned Fibers but with Airgap  (15 points)

Two optical fibers are connected together but with an air gap $d_g = 3$ micrometers; however, they are centerline aligned and the cleaved ends of each fiber are smooth and perpendicular. The cores of the two fibers both have a refractive index $n_1 = 1.50$ and the diameter $D$ of both cores is 50 micrometers. The magnitude of the partial reflection $r$ of light transmitted through the air gap interface is calculated using the classical Fresnel formula for normal incidence which is (also, note that $n = 1$ for air):

$$r = \left( \frac{n_1 - n}{n_1 + n} \right)^2$$

This equation is for one fiber core to air interface and for an air gap between two fibers being mated there are two interfaces, so it must be applied twice. How much loss does the $3 \, \mu m$ air gap introduce in the fiber splice? Express your answer in decibels (dB) where loss in dB is given by

$$\text{Fresnel Loss} = -10 \cdot \log_{10} (1 - r)$$

Problem 7  Two Aligned Fibers with Index Gel  (15 points)

Repeat the calculation from Problem 6 above but now with a gel replacing the air gap. The effective index of refraction of the gel is $n_{gel} = 1.44$. Note: This would be typical for a fiber splice.

(a) What is the loss (in dB) for the gel gilled splice of these two fibers?
(b) What fractional improvement does the gel give over the air gap case in Problem 6?