Lecture 5

http://www.wiretechworld.com/the-future-of-optical-fibres/
Linearly Polarized (LP) Modes

From Lecture 4:

- These *linearly polarized* (LP) modes, designated as $LP_{lm}$, are good approximations formed by exact modes TE, TM, HE and EH.

- The mode subscripts $l$ and $m$ describe the electric field intensity profile. There are $2l$ field maxima around the fiber core circumference and $m$ field maxima along the fiber core radial direction.

https://www.slideshare.net/tossus/waveguiding-in-optical-fibers
LP\textsubscript{01} and LP\textsubscript{11} Modes in the Core of an Optical Fiber

When light is launched into a fiber, modes are excited to varying degrees depending on the conditions of the launch — input cone angle, spot size, axial centration, etc. The distribution of energy among the modes evolves with distance as energy is exchanged between them. Energy can be coupled from guided to radiation modes by perturbations such as microbending and twisting of the fiber.

https://www.newport.com/t/fiber-optic-basics
The \textit{V} Number (or Fiber Parameter)

The \textit{V} number is an often-used dimensionless parameter in the context of step-index fibers. \textit{V} is given by

\begin{align*}
V &= \left( \frac{2\pi a}{\lambda} \right) \sqrt{n_1^2 - n_2^2} = \left( \frac{2\pi a}{\lambda} \right) NA = \left( \frac{2\pi a}{\lambda} \right) n_1 \sqrt{2\Delta}
\end{align*}

\begin{itemize}
  \item \textit{a} \quad \text{Radius of fiber core within the cladding}
  \item \textit{n}_1 \quad \text{Index of refraction of fiber core}
  \item \textit{n}_2 \quad \text{Index of refraction of cladding}
  \item \textit{\lambda} \quad \text{Wavelength of optical signal in vacuum}
  \item \textit{NA} \quad \text{Numerical Aperature}
  \item \textit{\Delta} \quad \text{Index difference between \textit{n}_1 and \textit{n}_2} \quad \Delta = \frac{n_1^2 - n_2^2}{2n_1^2}
\end{itemize}

https://www.rp-photonics.com/v_number.html
Propagation Constant vs. $V$-number for Different Modes

$n_2 k \leq \beta \leq n_1 k$

$V = 2.405$

Single mode range

The $V$ Number (or Normalized Frequency)

The $V$ number is a dimensionless parameter which is often used in the context of step-index fibers. It is defined as

$$V = \left(\frac{2\pi a}{\lambda}\right)\sqrt{n_1^2 - n_2^2} = \left(\frac{2\pi a}{\lambda}\right)NA$$

where $\lambda$ is the vacuum wavelength, $a$ is the radius of the fiber core, and $NA$ is the numerical aperture. The $V$ number can be interpreted as a kind of normalized optical frequency. (It is proportional to the optical frequency but rescaled depending upon waveguide properties.) It is relevant for various essential properties of a fiber:

- For $V$ values below $\approx 2.405$, a fiber supports only one mode per polarization direction (→ single-mode fibers).
- Multimode fibers can have much higher $V$ numbers. For large values, the number of supported modes of a step-index fiber (including polarization multiplicity) can be calculated approximately as

$$M \approx \frac{V^2}{2}$$
Optical Fiber Cut-Off Wavelength

The cut-off frequency of an electromagnetic waveguide is the lowest frequency for which a mode will propagate in it. In fiber optics, it is more common to consider the **cut-off wavelength**, the maximum wavelength that will propagate in an optical fiber or waveguide. The cut-off wavelength is given by

\[
\lambda_C = \left( \frac{2\pi a}{V_{lm}} \right) \sqrt{n_1^2 - n_2^2}
\]

Cut-off wavelength can also be defined as the wavelength below which multimode transmission starts.

\[
\lambda_C = \left( \frac{2\pi a}{2.405} \right) \times NA
\]

Single Mode Optical Fiber

Step Index Fiber

https://physics.stackexchange.com/questions/106477/graded-index-fiber?rq=1
Mode Field Diameter (MFD)

For SMF, the diameter of the circular region for the light intensity of $1/e^2 = 0.135$ of the core center light intensity is defined as mode field diameter (provided that the light intensity distribution in SMF is approximated by the Gaussian function).

$$MFD = 2w$$

http://en.optipedia.info/lsource-index/fiberlaser-index/fiber/parameters/mode-field/
Mode Field Diameter (MFD)

A different definition of Mode Field Diameter

Figure 2.31 Field amplitude distribution $E(r)$ of the fundamental mode in a single-mode fiber illustrating the mode-field diameter (MFD) and spot size ($\omega_0$)

From Figure 2.31 (page 61) in Senior.
Mode-Field Diameter is Dependent Upon Wavelength

- Mode-field intensity distribution can be measured directly by near-field imaging the fiber output.

https://www.slideshare.net/tossus/waveguiding-in-optical-fibers
Next Topic: **Dispersion in Optical Fibers**

Sections 3.8 to 3.12 in Senior (pages 105 to 140)
Also Section 3.13.2.
Digital Bit Rate

For no overlapping light pulses within an optical fiber, the digital bit rate \( (B_T) \) must be less than the reciprocal of the broadened pulse duration \( (2\tau) \). Hence,

\[
B_T \leq \frac{1}{2\tau} \quad \text{(approximately)}
\]

Note: Must distinguish between bit rate and bandwidth.
Conversion of bit rate $B_T$ (bps) to bandwidth $BW$ (Hz) depends upon the digital coding:

For Nonreturn-to-Zero: $B_T(\text{max}) = 2BW$

For Return-to-Zero: $B_T(\text{max}) = BW$

From: Figure 3.8 (page 107) in Senior.
Bandwidth-Length Product Is Most Useful Metric

The information carrying capability of an optical fiber is limited by the amount of pulse distortion at the receiving end of the fiber. Pulse broadening is proportional to the length of the fiber cable and thus bandwidth is inversely proportional to distance.

The Bandwidth-Length parameter is the most useful parameter to state the information capability of a transmission line (optical fiber).

\[ B_T \times L \]

A more accurate estimate for maximum bit rate \( B_T(max) \) is obtained if we assume a Gaussian pulse with standard deviation \( \sigma \)

\[ B_T(max) \approx \frac{0.2}{\sigma} \text{ [bits/sec]} \]

See Appendix B (pp. 1052-1053) of Senior
Example 3.5 (Page 109) of Senior

A multi-mode graded index fiber exhibits a pulse broadening of 0.1 microseconds (1 μs) over a distance of 15 km.

(a) Estimate the maximum possible bandwidth of this fiber link assuming not ISI.

The maximum possible optical bandwidth which is equivalent to the maximum Possible bit rate (for to return-to-zero pulses) without ISI is

\[ B_{opt} = B_T = \frac{1}{2\tau} = \frac{1}{0.2 \times 10^{-6} \text{ second}} = 5 \text{ MHz} \]

(b) The dispersion per unit length may be estimated using by dividing the total dispersion by the total length of the fiber link.

\[ \text{Dispersion} = \frac{0.1 \times 10^{-6} \text{ second}}{15 \text{ km}} = 6.667 \text{ ns/km} \]
Example 3.5 (continued)

(c) The bandwidth-length product can be determined in two ways. First, multiply the maximum bandwidth by its length.

\[ B_{\text{opt}} L = 5 \text{ MHz} \times 15 \text{ km} = 75 \text{ MHz} \times \text{km} \]

The other way is to use the dispersion per unit length and apply it to \( B_T \leq 1/2\tau \).

\[ B_{\text{opt}} L = \frac{1}{2\tau} = \frac{1}{2 \times 6.667 \times 10^{-9} \text{ [sec/km]}} = 75 \text{ MHz} \times \text{km} \]
Bit-Rate $\times$ Distance Product Evolution

https://www.researchgate.net/figure/System-capacity-per-fiber-in-optical-communication-systems_fig2_309307654
# Bandwidth × Distance Product (MHz × km)

Some commonly cited numbers

<table>
<thead>
<tr>
<th>Fiber</th>
<th>Bandwidth x Distance Product (MHz x km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size (μm)</td>
<td>Wavelength</td>
</tr>
<tr>
<td>9/125</td>
<td>SM</td>
</tr>
<tr>
<td>50/125</td>
<td>MM</td>
</tr>
<tr>
<td>62.5/125</td>
<td>MM</td>
</tr>
</tbody>
</table>

https://fiberopticnetwork.wordpress.com/tag/fibre-optic-bandwidth/
Pulse Broadening From Intermodal Dispersion

From: Figure 3.9 (page 108) in Senior.
Pulse Spreading & Attenuation Caused by Dispersion

https://www.researchgate.net/figure/Pulse-Spread-and-Attenuation-due-to-Dispersion_fig1_277014078
Dispersion Mechanisms in Optical Fibers

MMF = Multi-Mode Fiber; SMF = Single Mode Fiber

https://www.intechopen.com/books/current-developments-in-optical-fiber-technology/multimode-graded-index-optical-fibers-for-next-generation-broadband-access
Modal Dispersion (also called Intermodal Dispersion)

Modal Dispersion is a distortion mechanism occurring in multimode fibers, where signals spread in time because the propagation velocity of the signal is not the same for all modes.

Chromatic Dispersion

Chromatic Dispersion is the result of the different colors, or wavelengths, in a light beam arriving at their destination at slightly different times. Thus, the refractive index is a function of wavelength.

Waveguide Dispersion

Waveguide Dispersion is a result of the different refractive indexes of the core and cladding of an optical fiber. Some of the signal travels in the cladding, as well as the core. Hence, the structure of the optical fiber enters into the dispersion.
Dispersion Mechanisms in Optical Fibers – II

Polarization Mode Dispersion

Polarization Mode Dispersion (PMD) is a form of modal dispersion where two different polarizations of light, which normally travel at the same speed, now travel at different speeds due to their different polarizations.

Material Dispersion

Material Dispersion is one kind of Chromatic Dispersion. It is the variation of the propagation velocity through a transparent material with wavelength. It is called “material” dispersion to distinguish it from a similar effect called “waveguide” dispersion, which is a consequence of the process by which light is guided in a dielectric fiber.
Group Delay

- **Group delay** is the time required for a mode to travel down a fiber of length $L$.
- Total delay is the group delay per unit length along the fiber.
- The collection of all frequencies constitutes the signal’s waveform and the energy of the signal.
- Each spectral component travel independently of the others.
- **Group velocity** is the speed of the waveform (group of frequencies) as it travels along the fiber.

\[
\tau_{gr} = \frac{1}{L} = \frac{1}{v_{gr}} = \frac{1}{c} \frac{d\beta}{dk} = - \frac{\lambda^2}{2\pi c} \frac{d\beta}{d\lambda}
\]

$L = \text{distance traveled by the signal}$

$\beta = \text{propagation constant}; \ k = \frac{2\pi}{\lambda}$

Ref. Section 2.3.3, pp. 28 to 30 in Senior
Group Velocity

A packet of waves moves at the **group velocity** given by

\[ V_g = V_{gr} = \frac{\partial \omega}{\partial \beta} = \left( \frac{\partial \beta}{\partial \omega} \right)^{-1} = c \left( \frac{d\beta}{dk} \right)^{-1} \]

where

\[ \beta = n_1 \frac{2\pi}{\lambda} = n_1 \frac{\omega}{c} \]

The group velocity is the velocity of the energy transport of the optical signal propagating on the fiber.

The phase velocity of a single frequency is

\[ V_p = V_{ph} = \frac{\omega}{\beta} \]
Group Velocity

\[ V_{gr} = \frac{d \lambda}{d \beta} \frac{d \omega}{d \lambda} = \frac{d}{d \lambda} \left( n_1 \frac{2 \pi}{\lambda} \right)^{-1} \left( \frac{-\omega}{\lambda} \right) \]

\[ = \left( \frac{-\omega}{2 \pi \lambda} \right) \left( \frac{1}{\lambda} \frac{dn_1}{d\lambda} - \frac{n_1}{\lambda^2} \right)^{-1} \]

\[ = \left( n_1 - \frac{\lambda}{\lambda} \frac{dn_1}{d\lambda} \right) = \frac{c}{N_{gr}} \]

where \( N_{gr} \) is known as the group index of the fiber.

Ref. Section 2.3.3, pp. 28 to 30 in Senior
Modal Dispersion

Each mode has unique velocity of propagation

https://apps.lumerical.com/pic_circuits_optical_fiber.html
Modal Dispersion

The minimum time $T_{\text{min}}$ for a signal to travel distance $L$ within the fiber is

$$T_{\text{min}} = \frac{\text{distance}}{\text{velocity}} = \frac{L}{(c / n_1)} = \frac{Ln_1}{c}$$

The maximum time $T_{\text{max}}$ for a signal to travel distance $L$ within the fiber is

$$T_{\text{max}} = \frac{\text{distance}}{\text{velocity}} = \frac{Ln_1}{c \cdot \cos(\theta)}$$

Using Snell’s Law we can write

$$T_{\text{max}} = \frac{Ln_1^2}{c \cdot n_2}$$

Ref. Section 3.10.1, pp. 114 to 119 in Senior
The delay difference between meridional ray and the axial ray by subtraction,

\[ \delta T = T_{\text{max}} - T_{\text{min}} = \frac{\text{Ln}^2_1}{c \cdot n_2} - \frac{\text{Ln}_1}{c} = \frac{\text{Ln}^2_1}{c \cdot n_2} \left( \frac{n_1 - n_2}{n_2} \right) \approx \frac{\text{Ln}^2_1}{c \cdot n_2} \Delta \]

where delta \( \Delta \) is

\[ \Delta = \frac{n_1 - n_2}{n_1} \]

And

\[ \delta T = \frac{\text{Ln}_1}{c} \left( \frac{n_1 - n_2}{n_2} \right) \approx \frac{\text{Ln}_1}{c} \Delta \quad \text{or} \quad \delta T = \frac{L(NA)^2}{2n_1 c} \]

What we want to know is the rms pulse broadening \( \sigma_{MD} \)

\[ \sigma_{MD} = \sigma_s \approx \frac{\text{Ln}_1 \Delta}{2 \sqrt{3c}} = \frac{L(NA)^2}{4 \sqrt{3} n_1 c} \]
Example 3.8 (Modal Dispersion)

Example 3.8. A six-kilometer (6 km) optical link with multi-mode step-index fiber with core index of refraction = 1.500 and relative refractive index difference of 1% -- Find
(a) The delay difference between the lowest and fastest modes of the fiber link.

Solution: The delay difference is given by

\[ \delta T = \frac{L n_1 \Delta}{c} = \frac{(6 \times 10^3 \text{ m}) \times 1.500 \times 0.01}{3 \times 10^8 \text{ m/sec}} = 300 \text{ ns} \]

(b) Find the rms pulse broadening from modal dispersion over the fiber link of 6 km.

Solution:

\[ \sigma_{MD} = \frac{L n_1 \Delta}{2\sqrt{3}c} = \frac{1}{2\sqrt{3}} \frac{(6 \times 10^3 \text{ m}) \times 1.500 \times 0.01}{3 \times 10^8 \text{ m/sec}} = \frac{300}{3.4641} \text{ ns} = 86.6 \text{ ns} \]

(c) Next, find the maximum bit rate without errors on the fiber link.
Example 3.8 (continued)

Example 3.8.
(c) Next, find the maximum bit rate without errors on the fiber link. (assume only modal dispersion)

Solution: The maximum bit rate can be estimated in two ways. First, we may use the relationship between $B_T$ and the reciprocal of the delay $\tau$, namely,

$$B_T(\text{maximum}) = \frac{1}{2\tau} = \frac{1}{2\delta T} = \frac{1}{2 \times 300 \times 10^{-9} \text{ sec}} = 1.67 \text{ Mbps}$$

Or we can estimate it from the calculated rms pulse broadening from part (b).

$$B_T(\text{maximum}) \approx \frac{0.2}{\sigma_{MD}} = \frac{0.2}{86.6 \times 10^{-9} \text{ sec}} = 2.31 \text{ Mbps}$$

(d) Find the bandwidth-length product corresponding to (c) above. Assume return-to-zero pulses.

$$B_{opt} \times L = 2.31 \text{ MHz} \times 6 \text{ km} = 13.86 \text{ MHz} \cdot \text{km}$$