EE 443 Optical Fiber Communications
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Fall Semester

Lecture 14

Carrier Motion & Noise

http://www.wiretechworld.com/the-future-of-optical-fibres/
1. Avalanche carrier multiplication arises from impact ionization of energetic carriers hitting the lattice (these carriers gain high energy from the strong electric field within the depletion region)

2. The multiplication factor $M$ is defined as the ratio of the multiplied photocurrent divided by the primary unmultiplied photocurrent

3. Avalanche photodiodes are doped so that only part of their depletion layer exceeds the critical electric field $E_C$ where avalanche multiplication occurs

4. The ionization coefficients model the strength of the multiplication; coefficients are different for different semiconductors and increase in magnitude with increasing electric field within the depletion region

5. Avalanche multiplication is an inherently noisy process
Summary of Lecture 13 (continued)

6. High electric fields at junction edges lead to excessive leakage current (like dark current) and lower breakdown voltages – guard-band rings are used at junction edges to reduce edge degradation

7. The major advantage of the avalanche photodiode (APD) is the inherent gain from carrier multiplication leading to superior performance over PIN photodiodes for recovering low-level optical signals

8. APDs have faster response times than pn-junction and PIN photodiodes

9. However, APDs are more difficult to fabricate, possess higher noise, require higher reverse voltages, lower reliability and output a nonlinear current versus optical power input
Summary of Lecture 13 (continued)

10. Solar cells and photodiodes are similar; solar cells are operated without applied voltage and output power; photodiodes are reverse-biased and their output is current proportional to input optical power

11. Triple junction (and quad-junction) solar cells have been fabricated using multiple layers of different compound semiconductors which are much more efficient in collecting solar radiation

12. Typically photodiodes (of all structures) are followed immediately by transimpedance amplifiers (TIA) to provide isolation, gain and to convert photocurrent into an output voltage

13. Careful attention must be given to the packaging of photodiodes to provide biasing, environmental protection, temperature control and deliver adequate gain for follow-on signal processing
Thermal Motion of Electrons (or Holes) in a Material

2.1 Thermal Motion

- Zig-zag motion is due to collisions or scattering with imperfections & phonons in the crystal
- Net average thermal velocity is zero
- Mean time between collisions is $\tau_m \sim 0.1\text{ps}$

Modern Semiconductor Devices for Integrated Circuits (C. Hu)
Thermal of Carriers Using Kinetic Theory

Each carrier has an energy of $\frac{1}{2}k_B T$ per degree of freedom

$$\langle KE \rangle = \frac{3}{2}k_B T$$

$$\langle KE \rangle = \frac{1}{2}m^* \langle v^2 \rangle$$

$$\sqrt{\langle v^2 \rangle} = v_{rms} = \sqrt{\frac{3k_B T}{m^*}}$$

$$v_{rms} = v_{th} \approx 10^7 \text{ cm/sec}$$
Thermal Motion of Electrons (or Holes) in a Material

Charge carriers undergo collisions with both vibrating lattice atoms (i.e., phonons) and with ionized (charged) doping atoms in the lattice.

We can define a characteristic time constant due to the random collisions from the thermally induced motion. Let the mean free time between collisions be $\tau_c$. There will be a mean thermal velocity $v_{th}$.

The mean free path length between collisions, denoted by $\lambda$, is

$$\lambda = v_{th} \cdot \tau_c$$

If $\tau_c \approx 10^{-13}$ seconds, and

$v_{th} \approx 10^7$ cm/sec, then

$\lambda = 0.01 \, \mu$m or $10$ nm
Random Electron Motion With Superimposed Drift Motion

\[ \nu_{th} \approx 10^7 \text{ cm/sec} \]

\[ F = -qE \]

Drift Current in a Semiconductor (Drift Velocity)

\[ n = \text{number of charges} \, e \, \text{per unit volume} \]
\[ Q = qn(Ad) = \text{total mobile charge in length} \, d \, \text{of the conductor} \]
\[ t = \frac{d}{v_{\text{drift}}} = \text{time for this charge to sweep past the current measuring point.} \]

Average drift velocity of charge carriers

\[ I = \frac{Q}{t} = \frac{qn(Ad)}{d} \]

\[ I = qnAv_{\text{drift}} \]

http://hyperphysics.phy-astr.gsu.edu/hbase/electric/miccur.html
Drift Velocity in a Semiconductor Sample

Given a semiconductor doped n-type with a concentration of $1 \times 10^{18}$ cm$^{-3}$. The cross-section of the semiconductor sample is 0.01 cm$^2$.

Find the average drift velocity with a current $I$ of 1 ampere (equals a current density $J$ of 100 A/cm$^2$).

\[
J = \frac{I}{A} = 100 \text{ A/cm}^2 = qnv_{\text{drift}} = (1.602 \times 10^{-19} \text{ coul})(1 \times 10^{18} \text{ cm}^{-3})v_{\text{drift}}
\]

Therefore,

\[
v_{\text{drift}} = \frac{100 \text{ A/cm}^2}{(1.602 \times 10^{-19} \text{ coul})(1 \times 10^{18} \text{ cm}^{-3})} = 624 \text{ cm/sec}
\]

That is equivalent to about 20 feet per second. Compare this to the thermal velocity of carriers at approximately 10,000,000 cm/sec.
Mobility in a Semiconductor ($\rightarrow$ Drift Current)

The concept of drift velocity comes from the acceleration of a charged carrier from an applied electric field $E$. The force on the carrier from the Electric field is

$$\text{Force} = |F| = qE,$$

$$|v(t)| = at = \frac{qE}{m^*} t$$

On the average there is a collision Every $\tau_c$ seconds. The net velocity In the applied electric field is

$$v_{\text{average}} = \frac{qE}{2m^*} \tau_c = \frac{q\tau_c}{2m^*} E$$

Define the mobility $(\text{cm}^2/\text{V-sec})$ as

$$\mu_{n,p} = \frac{q\tau_c}{2m_{n,p}^*} \quad (\text{mobility})$$
Mobility in a Semiconductor (→ Drift Current)

Electrons:

\[ \nu_{\text{drift},n} = -\mu_n E \quad \text{and} \quad \nu_{\text{drift},p} = \mu_p E \]

http://conocimientoscarriertransport.blogspot.com/2010/06/carer-mobility-semiconductors.html

Saturated Drift Velocity in a Semiconductor

$v_{sat} = 10^7 \text{ cm/sec}$

$E_c = 10^4 \text{ V/cm}$

$E_c$ is the critical electrical field

Diffusion Currents in Semiconductors

**Diffusion** Current is a current in a **semiconductor** caused by the **diffusion** of charge carriers (holes and/or electrons). This is the current which is due to the transport of charges occurring because of non-uniform concentration of charged particles in a **semiconductor**.

https://www.youtube.com/watch?v=fqg_TcMoAZA
Fick’s First Law

\[ J = -D \frac{dC}{dx} \]

- The negative sign of equation signifies that diffusion occurs in a direction opposite to that of increasing concentration.
- That is, diffusion occurs in the direction of decreasing concentration of diffusant; thus, the flux is always a positive quantity.
- The diffusion coefficient, \( D \) it does not ordinarily remain constant.
- \( D \) is affected by concentration, temperature, pressure, solvent properties, and the chemical nature of the diffusant.
- Therefore, \( D \) is referred to more correctly as a diffusion coefficient rather than as a constant.

https://www.slideshare.net/arijabuhaniyeh/diffusion-physical-pharmacy
Diffusion Currents From Concentration Gradients

Concentration gradient

\[ J_{n,\text{diff}} = qD_n \frac{dn}{dx} \]

Why no negative sign?

http://britneyspears.ac/physics/diffusion/diffusion.htm
Diffusion Currents in Semiconductors

- Due to thermally induced random motion, mobile particles tend to move from a region of high concentration to a region of low concentration.

- Current flow due to mobile charge diffusion is proportional to the carrier concentration gradient.

- Diffusion current within a semiconductor consists of hole and electron components:

\[
J_{p,\text{diff}} = -qD_p \frac{dp}{dx} \quad J_{n,\text{diff}} = qD_n \frac{dn}{dx}
\]

\[
J_{\text{tot, diff}} = q(D_n \frac{dn}{dx} - D_p \frac{dp}{dx})
\]

Drift & Diffusion Currents in Semiconductors

\[ J_e = q \left( n \mu_e E + D_e \frac{dn}{dx} \right) \quad \text{Electrons} \]

\[ J_p = q \left( p \mu_p E - D_p \frac{dp}{dx} \right) \quad \text{Holes} \]
Noise in Optical Receivers

Note a resistor is used to bias the photodiode.

SMF = single-mode fiber

https://www.sciencedirect.com/topics/engineering/optical-receiver
White Gaussian Thermal Noise With Zero Mean

Gaussian PDF showing with $\mu = 0$ and the Std. Dev. to $\pm 3 \sigma$

https://commons.wikimedia.org/wiki/File:White_noise.svg
Johnson–Nyquist noise (Thermal noise, Johnson noise, or Nyquist noise) is the electronic noise generated by the thermal agitation of the charge carriers (usually the electrons) inside an electrical conductor at equilibrium, which happens regardless of any applied voltage.

\[
\begin{align*}
V_{\text{rms}}^2 &= \text{Mean square noise voltage} \\
I_{\text{rms}}^2 &= \text{Mean square noise current} \\
\Delta f &= \text{Noise bandwidth (Hz)} \\
k_B &= \text{Boltzmann's constant (1.381 × 10}^{-23} \text{ J/K)} \\
T &= \text{Temperature (Kelvin)} \\
R &= \text{Resistance (ohms)}
\end{align*}
\]

Thermal Noise Power and Noise Voltage of a Resistor

Calculate the thermal noise power available from any resistor at room Temperature (290 K) for a bandwidth of $\Delta f = 1$ MHz ($1 \times 10^6$ Hz). Also Calculate the corresponding noise voltage if $R = 50$ ohms.

Note: $kT = (1.381 \times 10^{-23} \text{ J/K})(290 \text{ K}) = 4.00 \times 10^{-21} \text{ J}$

1. Thermal noise power

$$P_n = kT\Delta f$$

$$P_n = (4.00 \times 10^{-21} \text{ J}) \times 1 \times 10^6 \text{ sec}^{-1}$$

$$P_n = 4.00 \times 10^{-15} \text{ W} \quad (W = \text{J/sec})$$

$$P_n = 10 \cdot \log_{10}\left(\frac{4.00 \times 10^{-12} \text{ mW}}{1 \text{ mW}}\right)$$

$$P_n = -114 \text{ dBm}$$

$P_n$ is independent of the value of $R$

2. Thermal noise voltage (rms)

$$v_n = \sqrt{4kTR\Delta f}$$

$$v_n = \sqrt{4 \times (4.00 \times 10^{-21} \text{ J}) \times 50 \Omega \times (10^6 \text{ sec}^{-1})}$$

$$v_n = \sqrt{8.01 \times 10^{-13} \text{ V}^2}$$

$$v_n = 0.895 \mu\text{V}$$
### Shot Noise

**Shot noise** or **Poisson noise** is a type of noise which can be modeled by a Poisson process. In electronics shot noise originates from the discrete nature of electric charge. Shot noise also occurs in photon counting in optical devices, where shot noise is associated with the particle nature of light.

**Mathematical Expression**

\[
\langle i_n^2 \rangle = I_{rms}^2 = 2qI_{DC}\Delta f
\]

**Examples**
- Poisson fluctuations of charge carrier number - eg arrival of charges at electrode in system induce charges on electrode
- Electrons/holes crossing potential barrier in diode or transistor
- Electron flow in vacuum tube

**Gaussian Distribution**

Current fluctuations in \( I_{DC} \) follow a Gaussian distribution.
Shot Noise

Avalanche Multiplication

Diode I-V characteristic

This is an empirical equation:

\[ M = \frac{1}{\left(\frac{V_r}{V_{brkd}}\right)^m}, \quad m \approx 3 \text{ to } 6 \]

http://www.learningaboutelectronics.com/Articles/What-is-a-zener-diode

https://docplayer.net/51269316-Downloaded-on-t12-48-44z.html
Avalanche noise is associated with reverse-biased junctions. Carriers in the junctions gain energies in a high electrical field and collide with the crystal lattice. If the energy gained between collisions is large enough, another pair of carriers (electron & hole) can be generated. Thus, the revised biased current can be multiplied. This is a random process and obviously a noise source. The intensity of the avalanche noise is usually much larger than other noise components.


https://electronics.stackexchange.com/questions/359251/why-is-zener-avalanche-noise-saw-tooth-shaped
## Summary of Thermal, Shot and Avalanche Noise

<table>
<thead>
<tr>
<th>Noise categories in electronic devices</th>
<th>Name</th>
<th>aka</th>
<th>Dominant Mechanism</th>
<th>Applies to</th>
<th>Dependence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Thermal</td>
<td>Johnson Nyquist</td>
<td>Random thermal carrier motion (analogy: Brownian motion); Always present for T &gt; 0 K</td>
<td>Resistance (or Conductance)</td>
<td>Temperature and resistance</td>
</tr>
<tr>
<td></td>
<td>Shot</td>
<td>Schottky Poisson</td>
<td>Due to discrete quantum nature of electronic charge carriers</td>
<td>Present when current flows across a barrier or boundary</td>
<td>DC current</td>
</tr>
<tr>
<td></td>
<td>Avalanche</td>
<td>----</td>
<td>Due to impact of charge carriers into lattice (phonons and charge impurities) where avalanche multiplication adds carriers</td>
<td>Strongly reverse biased pn-junctions</td>
<td>Electric field and Temperature</td>
</tr>
</tbody>
</table>
Photodetector Noise Including Amplification

We require a photodiode to faithfully detect weak optical signals in the presence of noise from the detector and subsequent circuitry. It is convenient to work with a parameter called the signal-to-noise ratio, $S/N$, and is defined at the output of an optical receiver as

$$\frac{S}{N} = \frac{\text{signal power from detector photocurrent}}{\text{detector's noise power + amplifier's noise power}}$$

The noise sources in the receiver arise from the photodetector noises resulting from the statistical nature of the photo-to-electron conversion process and thermal noise from the associated circuitry around the photodetector. Thus, we want high quantum efficiency $\eta$ in a photo-Diode to accomplish lowest $S/N$ or $SNR$. 
Photodiode Noise

We treat quantum noise as shot noise in our analysis. A photodiode produces a photocurrent $I_p$ and the noise generated by the photodiode is the shot noise of both the dark current $I_d$ and the photon generated photocurrent $I_p$, thus the mean-square noise current is

$$< i_s^2 > = 2qI_d \Delta f + 2qI_p \Delta f$$

This expression includes no contribution from the avalanche process. The signal-to-noise ratio (i.e., a power ratio) is simply

$$\frac{S}{N} = \frac{I_p^2}{< i_s^2 >} = \frac{I_p}{2q(I_d + I_p) \Delta f}$$

From: Section 9.2.5, pages 508 to 510; in Senior 3rd edition
Photodiode Noise

Recall the photocurrent $I_p$ can be written as $I_p = \frac{\eta P_0 q}{h\nu}$, then we can write the signal-to-noise power ratio, (Note: assuming $I_p >> I_d$)

$$\frac{S}{N} = \frac{\eta P_0 q}{h\nu 2q\Delta f} = \frac{\eta P_0}{2h\nu \Delta f}$$

Now the required incident optical power can be calculated for a specified signal-to-noise ratio.

**EXAMPLE 9.2:** An optical fiber system operating at a wavelength of 1 μm has a post-detection bandwidth of 5 MHz. Assume an ideal detector, calculate the incident optical power necessary to achieve a signal-to-noise ratio of 50 dB at the receiver.

From: Section 9.2.5, pages 508 to 510; in Senior 3rd edition
Example 9.2: Photodiode Noise

Solution: The incident power can be written as

\[ P_0 = \left( \frac{S}{N} \right) \frac{2hf \Delta f}{\eta} \]

For \( S/N = 50 \) dB, and using \( 10 \cdot \log_{10} \left( \frac{S}{N} \right) = 50 \), thus \( S/N = 10^5 \).

At \( \lambda = 1 \) µm, \( f = 3 \times 10^{14} \) Hz. For an ideal detector, the efficiency \( \eta = 1 \) and so the incident optical power is

\[ P_0 = \frac{10^5 \times 2 \times (6.626 \times 10^{-34}) \times (3 \times 10^{14}) \times (5 \times 10^6)}{1} \]

\[ P_0 = 198.8 \text{ nW} = 198.8 \times 10^{-6} \text{ mW}, \quad \text{and} \]

\[ P_0 = 10 \cdot \log_{10} (198.8 \times 10^{-6}) = -37.0 \text{ dBm} \]

From: Section 9.2.5, pages 508 to 510; in Senior 3rd edition
The **PN** and **PIN** Photodiode Receiver

The two main sources of noise in a photodiode without gain are dark current noise and quantum noise.

\[
\langle i_{Ts}^2 \rangle = 2q(I_d + I_p)\Delta f
\]

The other noise component that Senior refers to is background radiation noise which would add a current term to this expression. Hence,

\[
\langle i_{Ts}^2 \rangle = 2q(I_d + I_p + I_{bkgrd})\Delta f
\]

We will ignore background radiation noise in this class. However, we do need to account for the noise added by the bias resistor \( R_L \) used with the photodiode, namely

\[
\langle i_{RL}^2 \rangle = \frac{4kT\Delta f}{R_L}
\]

From: Section 9.3.1, pages 511 to 515; in Senior 3\textsuperscript{rd} edition
Example 9.3: **PIN Photodiode Receiver**

Example 9.3: Given a silicon PIN photodiode in a photo receiver with a quantum efficiency of 60% when operating at 0.9 µm. The dark current is 3 nA and the load resistor $R_L$ is 4,000 ohms. The incident optical power is 200 nW and the post detection bandwidth is 5 MHz. Compare the shot noise of the photodiode to the thermal noise of the load resistor $R_L$ at a temperature of 20 °C.

First, find $I_p$,

$$I_p = \frac{\eta P_0 q \lambda}{h} = \frac{\eta P_0 q \lambda}{hc}$$

$$I_p = \frac{0.6 \times (200 \times 10^{-9}) \times (1.602 \times 10^{-19}) \times (0.9 \times 10^{-6})}{(6.626 \times 10^{-34}) \times (3 \times 10^8)}$$

$$I_p = 88.0 \times 10^{-9} \text{ A} = 88.0 \text{ nA}$$

From: Example 9.3, page 512; in Senior 3rd edition
Example 9.3: PIN Photodiode Receiver

The total shot noise is

\[ <i_{Ts}^2> = 2q(I_d + I_p)\Delta f = 2 \times (1.62 \times 10^{-19}) \times ((3 + 87.1) \times 10^{-9}) \times (5 \times 10^6) \]

\[ <i_{Ts}^2> = 1.46 \times 10^{-19} \text{ A}^2 \]

and the root mean square (RMS) shot noise is

\[ \sqrt{<i_{Ts}^2>} = i_{Ts;rms} = 3.82 \times 10^{-10} \text{ A} \]

Next, consider the thermal noise of the resistor \( R_L \),

\[ <i_{RL}^2> = \frac{4kT\Delta f}{R_L} = \frac{4 \times (1.381 \times 10^{-23}) \times 293 \times (5 \times 10^6)}{4 \times 10^3} \]

\[ <i_{RL}^2> = 2.02 \times 10^{-17} \text{ A}^2 \]

Note: \( T = 20 ^\circ \text{C} \rightarrow 293 \text{ K} \)

From: Example 9.3, page 512; in Senior 3\textsuperscript{rd} edition
Example 9.3: *PIN Photodiode Receiver*

**Conclusion:**
The root mean square noise current of the resistor is

\[ \sqrt{\langle i^2_{RL} \rangle} = 4.50 \times 10^{-9} \text{ A} \]

Thus, the rms noise current from the resistor is almost twelve times larger that the rms noise current from the dark current plus the photodiode current generated by the optical input.

\[ \sqrt{\langle i^2_{Ts} \rangle} = 3.82 \times 10^{-10} \text{ A} \quad \text{versus} \quad \sqrt{\langle i^2_{RL} \rangle} = 4.50 \times 10^{-9} \text{ A} \]

Beware of the resistor noise!

**Added note:** This example did not include an amplifier following the diode.
Limiting Cases for SNR

- When the optical signal power is relatively high, then the shot noise power is much greater than the thermal noise power. In this case the SNR is called shot-noise or quantum noise limited.

- When the optical signal power is low, then thermal noise usually dominates over the shot noise. In this case the SNR is referred to as being thermal-noise limited.

https://www.slideshare.net/amitabhs5/optical-receivers-46652578?next_slideshow=1
Next up: Section 9.3.2 Receiver capacitance and bandwidth

https://www.skipprichard.com/ask-questions-to-improve-your-leadership/
Additional slides on quantum noise not covered in class (and not part of EE 443 in the Fall semester 2019)
Quantum Noise

The detection of photons is a discrete process because electron-hole pairs are created by the absorption of a photon. The probability of $P(n)$ of detecting $n$ photons in time period $\tau$ when it is expected that $z_m$ photons will be detected on average is

$$P(n) = \frac{z_m^n \cdot \exp(-z_m)}{n!} \quad \text{Poisson distribution}$$

Where $z_m$ is equal to the variance of the probability distribution. For a Poisson distribution the mean and the variance are equal. The electron rate $r_e$ generated by incident photons is (refer to Lecture 11, slide 34)

$$r_e = \frac{\eta P_o}{hf}$$

Thus, the number of electrons generated in time $\tau$ is equal to the average number of photons detected over this time period $z_m$, thus

$$z_m = \frac{\eta P_o \tau}{hf}$$

From: Section 9.2.3, pp. 504-505, in Senior 3rd ed.
Quantum Noise (continued)

The Poisson distribution for $z_m = 10$ and for $z_m = 1000$ are shown below. These represent the detection process for monochromatic coherent light.

Figure 9.1  Poisson distributions for $z_m = 10$ and $z_m = 1000$
Review: Poisson Distribution

Poisson Distribution Formula

\[ P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!} \]

where
\[ x = 0, 1, 2, 3, ... \]
\[ \lambda = \text{mean number of occurrences in the interval} \]
\[ e = \text{Euler’s constant} \approx 2.71828 \]

The mean value is \( \mu = \lambda \) (= \( z_m \) in book)
The variance is \( \sigma^2 = \lambda \)
The standard deviation is \( \sigma \)

https://www.onlinemathlearning.com/poisson-distribution.html
Distribution for Incoherent, Nonchromatic Photons

Incoherent light is emitted randomly by atoms and has no phase relationship between the photons. The Poisson distribution no longer holds, therefore we expect an exponential distribution to govern incoherent photons when averaged over the Poisson distribution. Although we do not prove this, the probability distribution is now

\[ P(n) = \frac{z_m^2}{(1 + z_m)^n} \quad \text{Bose–Einstein distribution} \]

The Bose-Einstein distribution governs random statistics of thermal light (this is the spectrum of blackbody radiation).

The Bose-Einstein probability distribution is plotted on the next slide.

From: Section 9.2.3, pp. 504-505, in Senior 3rd ed.
Probability Distribution for Incoherent Light

The statistical distribution for incoherent light is illustrated below. Parameter $z_m$ is the mean value.

\[ P(n) = \frac{z_m^2}{(1+z_m)^n} \]

*Bose–Einstein distribution*

**Figure 9.2** Probability distributions indicating the statistical fluctuations of incoherent light for $z_m = 10$ and $z_m = 1000$

From: Section 9.2.3, Figure 9.2, p. 505, in Senior 3rd ed.
Noise Calculation Example

**Example 6.8** An InGaAs *pin* photodiode has the following parameters at a wavelength of 1300 nm: \( I_D = 4 \text{ nA}, \eta = 0.90, R_L = 1000 \Omega \), and the surface leakage current is negligible. The incident optical power is 300 nW (−35 dBm), and the receiver bandwidth is 20 MHz. Find the various noise terms of the receiver.

**Solution:**

(a) First, we need to find the primary photocurrent. From Eq. (6.6),

\[
I_p = \frac{\eta q I_b}{h} = \frac{\eta q I}{h}
\]

\[
= \frac{(0.90)(1.6 \times 10^{-19} \text{ C})(1.3 \times 10^{-6} \text{ m})}{(6.625 \times 10^{-34} \text{ J} \cdot \text{s})(3 \times 10^8 \text{ m/s})}
\]

\[
= 0.282 \mu \text{A}
\]

(b) From Eq. (6.13), the mean-square shot noise current for a *pin* photodiode is

\[
\langle i_{\text{shot}}^2 \rangle = 2q I_p B_e
\]

\[
= 2(1.6 \times 10^{-19} \text{ C})(0.282 \times 10^{-6} \text{ A})(20 \times 10^6 \text{ Hz})
\]

\[
= 1.80 \times 10^{-18} \text{ A}^2
\]

or \( \sqrt{\langle i_{\text{shot}}^2 \rangle} \approx 1.34 \text{ nA} \)

(c) From Eq. (6.14), the mean-square dark current is

\[
\langle i_{DB}^2 \rangle = 2q I_D B_e
\]

\[
= 2(1.6 \times 10^{-19} \text{ C})(4 \times 10^{-9} \text{ A})(20 \times 10^6 \text{ Hz})
\]

\[
= 2.56 \times 10^{-20} \text{ A}^2
\]

or \( \sqrt{\langle i_{DB}^2 \rangle} \approx 0.16 \text{ nA} \)

(d) The mean-square thermal noise current for the receiver is found from Eq. (6.17) as

\[
\langle i_T^2 \rangle = \frac{4k_B T}{R_L} B_e
\]

\[
= \frac{4(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K})}{1 \text{ k}\Omega} B_e
\]

\[
= 323 \times 10^{-18} \text{ A}^2
\]

or \( \sqrt{\langle i_T^2 \rangle} \approx 18 \text{ nA} \)

Thus for this receiver the rms thermal noise current is about 14 times greater than the rms shot noise current and about 100 times greater than the rms dark current.

[https://slideplayer.com/slide/8129640/](https://slideplayer.com/slide/8129640/)
**Quantum Limit to Detection**

\[ P_r(n) = \bar{N}^n \frac{e^{-\bar{N}}}{n!} \]

- Ideal photo detector having unity quantum efficiency and no dark current.
- No e-h pair generated in absence of optical pulse. '0'
- Possible to find minimum received optical power required for specific BER performance in digital system.
- Called Quantum limit.

https://www.slideshare.net/MadhumitaTamhane/optical-fiber-communication-part-3-optical-digital-receiver
Quantum Limit to Detection

\[ P_r(n) = \frac{N^n e^{-N}}{n!} \]

- Optical pulse of energy \( E \) falls on photo detector in time interval \( \tau \).
- During transmission signal if too low to generate any e-h pair and detected as 0.
- Then for error probability \( P_r(0) \), there exists a minimum energy \( E \) at wavelength \( \lambda \), to be detected as 1.
- Probability that \( n=0 \) electrons are emitted in interval \( \tau \):

\[ P_r(0) = e^{-N} \]

https://www.slideshare.net/MadhumitaTamhane/optical-fiber-communication-part-3-optical-digital-receiver
Digital fiber optic link operating at wavelength 850nm requires maximum BER of $10^{-9}$. Find quantum limit and minimum incident power $P_0$ that must fall on photo detector, to achieve this BER at data rate of 10Mbps for simple binary level signaling system. Quantum efficiency is 1.

Solution – for maximum BER,--

\[ P_r(0) = e^{-\bar{N}} = 10^{-9} \]

\[ \bar{N} = 9 \ln 10 = 20.7 = 21. \]

\[ E = 20.7 \frac{hv}{\eta} \]

https://www.slideshare.net/MadhumitaTamhane/optical-fiber-communication-part-3-optical-digital-receiver
QUANTUM LIMIT TO DETECTION - PROBLEM

- Minimum incident power that must fall on photodetector $P_o$ --- $E = P_o \tau$
- Assuming equal number of 0 and 1, $1/\tau = B/2$

$$P_o = 20.7 \frac{hcB}{2\lambda}.$$  
$$= \frac{20.7(6.626 \times 10^{-34} J \cdot s)(3.0 \times 10^8 m/s)(10 \times 10^6 \text{ bits/s})}{2(0.85 \times 10^{-6} m)}$$  
$$= 24.2 \text{ pW}$$

or, when the reference power level is one milliwatt, 

$$P_o = -76.2 \text{ dBm}$$

https://www.slideshare.net/MadhumitaTamhane/optical-fiber-communication-part-3-optical-digital-receiver