EE 443 Optical Fiber Communications
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Fall Semester

Lecture 20

http://www.wiretechworld.com/the-future-of-optical-fibres/
Summary of Lecture 19

1. Optical nodes require components such as multiplexers, demultiplexers, optical add/drop multiplexers, reconfigurable optical add/drop multiplexers, and cross-connect switches.

2. Opto-electronic integrated circuits (OEIC) use planar and strip waveguide structures to carry optical signals.

3. Splitters are constructed using Y-junction path splitters and combiners are the inverse operation of splitters.

4. Splitters are a critical part of passive optical networks (called PONs).

5. Optical couplers are of two types: (1) core interaction type and (2) surface interaction type.

6. Couplers are made with n input ports and m output ports.

7. Couplers work by light scattering allowing for optical energy to bleed from one fiber core to an adjacent fiber core.

8. Couplers can be operated as switches, 3-dB splitters and fractional splitters where the splitting ratio controls the sampling of the main signal.

9. Key coupler parameters include the insertion loss, excess loss and the crosstalk loss (all usually expressed in decibels).
Summary of Lecture 19 (continued)

10. Electro-Optical effects (E-O effects) covered in Lecture 19 were the Pockels effect (a linear change in the index of refraction with an applied electric field) and the Kerr effect (a quadratic change in the index of refraction with an applied electric field).

11. Applications for E-O devices include scanning devices, phase modulators, polarization shifters, intensity modulators (Mach-Zehnder interferometers) and switches.

12. Electrodes are used on E-O materials in both longitudinal & transverse configurations to allow the internal electric field to be set by an applied voltage.

13. A Pockets cell phase modulator allows for a phase shift \( \phi = \phi_0 - \pi (V/V_\pi) \) where \( V \) is the externally applied voltage and \( V_\pi \) is the half-wave voltage.

14. A commonly used E-O material is lithium niobate (LiNbO\(_3\)) and is capable of being modulated very rapidly (up to frequencies approaching 100 GHz).

15. Splitting an optical signal into two branches and applying the E-O effect to one of the branches, and thereafter recombining the two branches allows for a Mach-Zehnder interferometer to be formed.
16. An E-O prism can be made using a wedge-shaped structure with electrodes attached to the ends allows for a voltage-controlled bend angle of refracted light. This can be used to make an optical multiplexer of demultiplexer.
Eye Patterns (Eye Diagram) for Digital Signals

In telecommunication, an eye pattern, also known as an eye diagram, is an oscilloscope display in which a digital signal from a receiver is repetitively sampled and applied to the vertical input, while the data rate is used to trigger the horizontal sweep.

Interpreting the Eye Diagram

https://www.ques10.com/p/5873/describe-the-eye-diagram-as-applicable-to-optical-/)
An eye diagram is used in electrical engineering to get a good idea of signal quality in the digital domain.

As the word ‘pseudo’ suggests, pseudo-random numbers are not random in the way you might expect, at least not if you're used to dice rolls or lottery tickets. Essentially, PRNGs are algorithms that use mathematical formulae or simply precalculated tables to produce sequences of numbers that appear random.

PRNGs are efficient, meaning they can produce many numbers in a short time, and deterministic, meaning that a given sequence of numbers can be reproduced at a later date if the starting point in the sequence is known.

These will be used to generate pseudo-random bit binary sequences.

https://www.random.org/randomness/
Pseudo-Random Binary Signal

A random binary signal is a random process that can assume one of two possible values at any time. A simple method of generating a random binary signal is to take Gaussian white noise, filter the noise for the desired spectra and then convert the noise to a binary signal by taking the sign of the filtered signal.

Oscilloscope with Memory for Displaying Multiple Traces

PRBS (pseudo-random bit stream) is generated for testing BER.

Display window of oscilloscope – trigger sets the beginning of each trace or window.

Data stream of pulses

Project all frames onto display.

Trigger oscilloscope on clock edges for all traces.
Bit Error Rate (BER) Measurement

\[
\text{BER} = \frac{\text{Number of bit errors}}{\text{Total number of bits transmitted}}
\]

How many bits in the PRBS bit pattern stream?

The lowest \(\text{BER}_{\text{min}}\) to be measured should have a PRBS length equals \(3 \times (1/\text{BER}_{\text{min}})\). Example: For \(\text{BER}_{\text{min}} = 10^{-9}\), then length > \(3 \times 10^9\) bits.
Example Eye Diagram

Generated binary waveform, which is subjected to additive noise and how it is displayed on an oscilloscope.

https://www.osapublishing.org/oe/fulltext.cfm?uri=oe-21-21-25197&id=269074
Eye Diagram Degradation As Function of Distance

Fiber loss is, of course, an important design issue, as it dictates the repeater spacing of a long-haul, light-wave system. Another design issue is fiber dispersion, which leads to broadening of individual pulses inside the fiber.

https://optiwave.com/resources/applications-resources/lightwave-system-components/
Characterizing Jitter in a Digital Data Stream

Bit Error Rate and Bit Error Ratio

In digital transmission, the number of **bit errors** is the number of received bits of a data stream over a communication channel that have been altered due to noise, interference, distortion or bit synchronization errors.

The **bit error rate** (BER) is the number of bit errors per unit time. The **bit error ratio** (also BER) is the number of bit errors divided by the total number of transferred bits during a studied time interval.
Intersymbol Interference (ISI)

ISI is unwanted interference from adjacent (usually previous) symbols.

ISI is caused by dispersion and other non-ideal transmission in channels.
Examples: Bit Error Rates in a Communication System

![Bit Error Rate vs Eb/No for various modulation schemes](https://www.embedded.com/wp-content/uploads/uploadedimages-semiconductors-ti-msp430-analogcombos.jpg)

\[ E_b/N_0 \text{ (dB)} \]
Advantages of Digital Over Analog For Communications

1. Digital is more robust than analog to noise and interference†
2. Digital is more viable to using regenerative repeaters
3. Digital hardware more flexible by using microprocessors and VLSI
4. Can be coded to yield extremely low error rates with error correction
5. Easier to multiplex several digital signals than analog signals
6. Digital is more efficient in trading off SNR for bandwidth
7. Digital signals are easily encrypted for security purposes
8. Digital signal storage is easier, cheaper and more efficient
9. Reproduction of digital data is more reliable without deterioration
10. Cost is coming down in digital systems faster than in analog systems and DSP algorithms are growing in power and flexibility

† Analog signals vary continuously and their value is affected by all levels of noise.
Gaussian Noise and Decision making

https://www.sciencedirect.com/topics/engineering/digital-transmission-system
Binary Transmission With AWGN

PDF:
\[ p(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ -\left[ \frac{(x - \mu)^2}{2\sigma^2} \right] \right\} \]  \hspace{1cm} (12.5)

\[ \mu = \text{standard deviation} \]
\[ \sigma = \text{mean value} \]

From: Senior, Optical Fiber Communication, 3rd ed.; Figure 12.37, p. 716
Probability of Error Occurring

Signals greater than the decision threshold $D$ are called a one and signals below $D$ are called a zero. In the presence of noise errors may occur. We define the following probabilities:

(1) Probability that a signal is transmitted as a 1, but is received as a 0 is $P(0|1)$
(2) Probability that a signal is transmitted as a 0, but is received as a 1 is $P(1|0)$

These are the two shaded areas in the prior slide.

Let probability that a 1 is transmitted be $P(1)$ and probability that a 0 is transmitted be $P(0)$, then the probability of an error occurring be $P(e)$

$$P(e) = P(1) \cdot P(0|1) + P(0) \cdot P(1|0)$$  (12.6)

From: Senior, Optical Fiber Communication, 3rd ed.; Section 12.6.3, pp. 715-725
Signal and Noise Currents vs Decision Current

Now we consider the currents in the signal and noise upon the Signal. Let

- Signal current is \( i_{\text{sig}} \)
- Noise current is \( i_N \)
- Decision current is \( i_D \)

If an any time when a binary 1 is transmitted the noise current is negative such that:

then the resulting current \( i_{\text{sig}} + i_N \) will be less than \( i_D \) and an error will occur. The corresponding probability of a transmitted 1 will be received as a 0 is written as

\[
P(0 \mid 1) = \int_{-\infty}^{i_D} p(i, i_{\text{sig}}) \, di \quad (12.8)
\]

From: Senior, Optical Fiber Communication, 3rd ed.; Section 12.6.3, pp. 715-725
Probability $P(0 \mid 1)$

$$p_1(x) = p(i, i_{\text{sig}}) = \frac{1}{\sqrt{2\pi} \sqrt{i_N^2}} \exp \left\{ - \frac{(i - i_{\text{sig}})^2}{2i_N^2} \right\} = Gsn \left[ i, i_{\text{sig}}, \sqrt{i_N^2} \right]$$

Where $i$ is the actual current, $i_{\text{sig}}$ is the peak signal current during the a binary 1 (this corresponds to the peak photodiode current $I_p$ when only a signal component is present) and $\sqrt{i_N^2}$ is the mean-square noise current.

Upon substituting we get

$$P(0 \mid 1) = \int_{-\infty}^{i_D} Gsn \left[ i, i_{\text{sig}}, \sqrt{i_N^2} \right] di \quad (12.11)$$

The probability that a binary 1 will be received when a 0 is transmitted is the probability that the received current will be greater than $i_D$ at some time during the 0-bit interval.
Probability $P(0 \mid 1)$

$$P(1 \mid 0) = \int_{i_D}^{\infty} p(i, 0)\,di$$  \hspace{1cm} (12.12)

Assuming the mean-square noise current in the zero state is equal to the mean-square noise current in the one state, namely $\sqrt{i_N^2}$, and for a zero bit $i_{\text{sig}} = 0$, then we have

$$p_0(x) = p(i, 0) = \frac{1}{\sqrt{2\pi} \sqrt{i_N^2}} \exp\left\{-\left[\frac{(i-0)^2}{2\sqrt{i_N^2}}\right]\right\} = Gsn\left[i, 0, \sqrt{i_N^2}\right]$$  \hspace{1cm} (12.13-14)

Hence, we may write by substituting,

$$P(1 \mid 0) = \int_{i_D}^{\infty} Gsn\left[i, 0, \sqrt{i_N^2}\right]\,di$$  \hspace{1cm} (12.15)

Problem! How do we evaluate these integrals?
Evaluating Integrals With a Gaussian Distribution

The integrals (12.11) and (12.15) are not readily evaluated. They are commonly written in terms of the error function (erf). The error function is defined as

\[
\text{erf}(u) = \int_0^u \frac{2}{\sqrt{\pi}} \exp(-z^2)dz
\]  

(12.16)

And the complementary error function (erfc) is

\[
\text{erfc}(u) = 1 - \text{erf}(u) = \int_u^\infty \frac{2}{\sqrt{\pi}} \exp(-z^2)dz
\]  

(12.17)

Hence,
Plot of the Error Function

\[
erf(x) = \frac{1}{\sqrt{\pi}} \int_{-x}^{x} e^{-t^2} \, dt = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-t^2} \, dt.
\]

https://en.wikipedia.org/wiki/Error_function
Plot of the Complementary Error Function

\[
erfc(x) = 1 - \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2) \, dt
\]

http://www.cplusplus.com/reference/cmath/erfc/
Values for Error Function & Complementary Error Function

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https://en.wikipedia.org/wiki/Q-function
The Q-Function

In statistics, the Q-function is the tail distribution function of the standard normal distribution.

Definition:

\[ Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{u^2}{2}\right) \, du. \]

Expressed as function of \( \text{erf}(x) \) and \( \text{erfc}(x) \):

\[
Q(x) = \frac{1}{2} \left( \frac{2}{\sqrt{\pi}} \int_{x/\sqrt{2}}^\infty \exp(-t^2) \, dt \right) \\
= \frac{1}{2} \left( 1 - \frac{1}{2} \text{erf}\left(\frac{x}{\sqrt{2}}\right) \right) - \text{or-} \\
= \frac{1}{2} \text{erfc}\left(\frac{x}{\sqrt{2}}\right).
\]

https://en.wikipedia.org/wiki/Q-function
Using $P(0|1)$ and $P(1|0)$ to Express $P(e)$

$$P(0|1) = \frac{1}{2} \left[ 1 - \text{erf} \left( \frac{i_{\text{sig}} - i_D}{\sqrt{2} \sqrt{i_N^2}} \right) \right] = \frac{1}{2} \text{erfc} \left( \frac{i_{\text{sig}} - i_D}{\sqrt{2} \sqrt{i_N^2}} \right)$$

(12.18)

$$P(1|0) = \frac{1}{2} \text{erfc} \left( \frac{|0 - i_D|}{\sqrt{2} \sqrt{i_N^2}} \right) = \frac{1}{2} \text{erfc} \left( \frac{|-i_D|}{\sqrt{2} \sqrt{i_N^2}} \right)$$

(12.19)

Assuming a binary code (equal numbers of 1s and 0s), then the net probability of error is one-half the two shaded areas shown on Slide 16; therefore,

$$P(e) = \frac{1}{2} (P(0|1) + P(1|0))$$

(12.20)

$$P(e) = \frac{1}{2} \left[ \frac{1}{2} \text{erfc} \left( \frac{i_{\text{sig}} - i_D}{\sqrt{2} \sqrt{i_N^2}} \right) + \frac{1}{2} \text{erfc} \left( \frac{|-i_D|}{\sqrt{2} \sqrt{i_N^2}} \right) \right]$$

(12.21)

From: Senior, Optical Fiber Communication, 3rd ed.; Section 12.6.3, pp. 715-725
Error Probability $P(e)$

We may simplify (12.21) by setting the threshold decision level between zero current and peak current, that is, $i_D = (1/2)i_{\text{sig}}$

\[
P(e) = \frac{1}{2} \left[ \frac{1}{2} \operatorname{erfc} \left( \frac{i_{\text{sig}} / 2}{\sqrt{2} \sqrt{i_N^2}} \right) + \frac{1}{2} \operatorname{erfc} \left( \frac{-i_{\text{sig}} / 2}{\sqrt{2} \sqrt{i_N^2}} \right) \right]
\]

\[
P(e) = \frac{1}{2} \left[ \operatorname{erfc} \left( \frac{i_{\text{sig}}}{2 \sqrt{2} \sqrt{i_N^2}} \right) \right]
\]  

(12.22)

The electrical SNR at the detector may be written in terms of the peak signal power to the rms noise power (mean-square noise current) as:

\[
\frac{S}{N} = \frac{i_{\text{sig}}^2}{\langle i_N^2 \rangle}
\]  

(12.23)

The angle brackets denote a time average.

From: Senior, Optical Fiber Communication, 3rd ed.; Section 12.6.3, pp. 715-725
Probability of Error Expressed as a Function of SNR

\[
P(e) = \frac{1}{2} \left[ \text{erfc} \left( \frac{\sqrt{\text{SNR}}}{2\sqrt{2}} \right) \right]
\]

(12.24)

From: Senior, Optical Fiber Communication, 3rd ed.; Section 12.6.3, pp. 715-725
Bit Error Rate As a Function of SNR

From the plot on the prior slide one can now generate a BER “waterfall” plot equating SNR to BER.

Optical signal-to-noise ratio $i_{\text{sig}} / \sqrt{\langle i_N^2 \rangle}$ dB

Electrical signal-to-noise ratio $i_{\text{sig}}^2 / \langle i_N^2 \rangle$ dB

From: Senior, Optical Fiber Communication, 3rd ed.; Section 12.6.3, pp. 715-725