EE 443 Optical Fiber Communications
Dr. Donald Estreich
Fall Semester

Lecture 3

http://www.wiretechworld.com/the-future-of-optical-fibres/
Summary of Lecture 2  (Fall Semester 2020) – I

1. Telecommunication Networks: Interconnecting nodes that exchange data
2. In telecommunication Networks require (1) High data capacity, (2) Scalable, (3) Deliver new & flexible services, (4) Reliable & secure, and (5) Affordable.
3. Local Area Network (LAN) is a small-area network often used to connect computers to peripheral devices.
4. Metropolitan Area Network (MAN) is larger than a LAN and often spans a large campus or city.
5. Wide Area Network (WAN) is the largest of the networks and spans multiple states and countries.
6. **First Generation** of optical networks: Networks that simply included optical links (point-to-point links) as replacements to copper wire or cable.
Summary of Lecture (Fall Semester 2020) – II

7. Single-link lightpath consists of (1) optical source, (2) optical fiber channel, and (3) optical detector for receiving the signal.

8. In a single-link lightpath the optical source is often an LED or a semiconductor laser diode; the optical detector is either a semiconductor PIN diode or APD diode.

9. Second Generation of optical networks: Networks that in addition to optical lightpath links include routing and switching capability by using OLT, OADM and OXC elements.

10. Multiplexer techniques: Generally Time Division Multiplexing (TDM), Frequency Division Multiplexing (FDM) and Wavelength Division Multiplexing (WDM).

11. A Multiplexer is device where multiple analog or digital signals are combined into a signal stream over a shared medium.
Summary of Lecture (Fall Semester 2020) – III

12. A Demultiplexer is a device performing the inverse operation of multiplexing; each signal on the shared medium is split out into individual links.

13. A TDM Multiplexer works by interleaving bytes from multiple input lines onto an output line while cycling through all inputs over and over.

14. A WDM Multiplexer works by directing all the inputs onto the output line where each input has its own distinct wavelength.

15. An Optical Line Terminal (OLT) is the endpoint hardware device in a passive optical network (PON). An OLT has two functions: Converting the standard signals used by a service provider to the frequency and framing used by the PON system.
Summary of Lecture (Fall Semester 2020) – IV

16. An Optical Add/Drop Multiplexer is a wavelength management device which provides for adding wavelengths and removing wavelengths while passing all other wavelengths through to the output.

17. An Optical Cross-Connect is a switching device providing for the selection of optical signals to be routed to selectable outputs.

18. Network Topologies: Bus, Mesh, Ring and Star. Other topologies such as hybrid and tree topologies can be defined.

19. Switching in networks – there are three basic switching approaches in networking. These are circuit-switched, message-switched and packet-switched.

20. Circuit-Switched Network – Wires and switches are connected to form a dedicated connection to each communication. An example is the Public Switched Telephone Network (PSTN).
21. Packet-Switched Network – Message is sent in packets and smart routers send these packets onto their final destinations. Many paths are available to each router which selects the best path. At their destination, the packets are assembled in proper order to serve the application. This is also called distributed communication.

### Summary of Lecture (Fall Semester 2020) – VI

22.

<table>
<thead>
<tr>
<th>Circuit-Switched “Connection-oriented”</th>
<th>Packet-Switched “Connectionless”</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bandwidth</strong> guaranteed once connected</td>
<td><strong>Bandwidth</strong> is dynamically allocated (aka “adaptive routing”)</td>
</tr>
<tr>
<td>Wastes <strong>Network Bandwidth</strong></td>
<td>Better use of <strong>Network Bandwidth</strong></td>
</tr>
<tr>
<td><strong>Network Capacity</strong> not affected by other traffic; but limited # connections</td>
<td><strong>Network Capacity</strong> independent of concurrent transmission of packets</td>
</tr>
<tr>
<td>Each connection is “dedicated”</td>
<td>Packet forwarding can use any route in the network (“store &amp; forward”)</td>
</tr>
<tr>
<td>Switches perform connections to establish dedicated circuit path for each communication session</td>
<td>Routers and protocols determine the packet routes and adjust to network conditions (“forwarding on the fly”)</td>
</tr>
<tr>
<td>If a connection fails a new connection must be re-established to recover</td>
<td>If one router fails the network still functions by bypassing that router</td>
</tr>
<tr>
<td><strong>Cost</strong> by connection time</td>
<td><strong>Cost</strong> by number of packets</td>
</tr>
</tbody>
</table>

Today both network categories now handle both voice and data.
23. Optical network architecture: Long-haul networks (40 to 100 Gbps; > 1000 km) interface with metro or regional networks (1 to 40 Gbps; < 1000 km) fan out to access networks (< 1 Gbps; < 10 km).

24. Access network architecture is commonly a hub (central office) connecting via feeder links to remote nodes that deliver services to customers via distribution links.

25. Examples of Internet access networks are digital subscriber lines (DSL), cable modems, satellite links, and optical fiber links. Sometimes cellular wireless is included.

26. Broadband services refers to the delivery of high-speed Internet that is always on and much faster than the older “dial-up” access.

27. A **passive optical network (PON)** is a fiber-optic telecommunications technology for delivering broadband network access to end-customers.
Summary of Lecture (Fall Semester 2020) – VIII

28. Ethernet is an older (IEEE 802.3 in 1983) and widely used technology to connect computers together in a LAN (or MAN, etc.). Originally was 10 Mbps, now Ethernet reaches data rates up to 100 Gbps.

29. SONET/SDH are standardized protocols that transfer multiple digital bit streams synchronously over optical fiber. It carries many signals of different bit rate capacities through a synchronous, flexible, optical hierarchy.

30. The SONET basic signal rate is 51.84 Mbps (aka STS-1) and for SDH it is 155.52 Mbps.

31. Fibre Channel (FC) is a high-speed data transfer protocol (running at 1, 2, 4, 8, 16, 32, and 128 gigabit per second rates) providing in-order, lossless delivery of raw block data; primarily used to connect computer data storage to servers. Fibre Channel is used in storage area networks (SAN) in commercial data centers.
32. The Optical Transport Network is a digital wrapper technology providing a network-wide framework that adds SONET/SDH-like features to WDM equipment. It creates a transparent, hierarchical network designed for use on both WDM and TDM devices.
Physical Construction of an Optical Fiber

Buffer jacket materials:
- Fluoropolymers
- Polyvinylidene fluoride (Kyner)
- Polytetrafluoroethylene (Teflon)

Coating materials:
- Acrylate
- Fluoroacrylate (medical)
- Silicone
- Polyimide (aerospace)

Core (silica)

Cladding (silica)

Aramid yarn (Kevlar®)

https://networksmania.wordpress.com/topics/transmission-media-2/fiber-optic-2/
An optical fiber is a single, hair-fine filament drawn from molten silica glass. Silica-based glass fibers typically exhibit a loss of 0.3 – 0.4 dB/km at 1550 nm transmission.

Comparing an Optical Fiber to a Human Hair Strand

[Image of optical fiber and human hair]

https://www.wired.com/story/corning-pure-glass-fiber-optic-cable/
Common wavelengths include 850 nm, 1300 nm, 1310 nm and 1550 nm.


https://www.newport.com/t/fiber-optic-basics
Loss Mechanisms For Silica Optical Fiber

Figure 3.3, page 93, of Senior, 3rd ed.

https://www.quora.com/Whats-the-lowest-frequency-that-can-be-transmitted-through-optical-fiber
Optical Fiber Windows for Four Generations of Fiber Systems

Fiber Loss Improvements

1. Reducing the impurity level (< 1 part per billion – ppb) in the manufacturing process and

2. Reducing macrobending/microbending losses through the manufacturing/cabling/installation processes.

http://www.olsontech.com/mr_fiber/fiber-history.htm
Transmission Line Loss (Attenuation)

https://community.fs.com/blog/understanding-loss-in-fiber-optic.html
Common Defects in Fused Silica

The origin of absorption in optical fibers

Schematic of common defects in fused silica glass [3]

https://netl.doe.gov/sites/default/files/event-proceedings/2017/crosscutting/Posters/2017_FinalPoster01_FE0027891_VirginiaTech.PDF
Loss Mechanisms in Optical Fiber

• **Extrinsic Impurity Ions Absorption**: Extrinsic impurity ions absorption from the presence of metallic ions (e.g., Cr$^{3+}$, Cu$^{2+}$, Fe$^{2+}$) and the OH$^{-}$ ion from dissolved water in glass.

• **Rayleigh Scattering**: It is the primary type of linear scattering. Caused by inhomogeneities produced during the fiber manufacturing process. For example, glass composition fluctuations cause fluctuations in the material refractive index and density fluctuations. Rayleigh scattering accounts for > 95% of the attenuation in optical fiber.

Transmission Line (Optical Fiber) Attenuation

Light power decreases exponentially as it travels along a fiber. Let $P(z)$ be the power along a fiber with an initial power $P(0)$ at the beginning of the fiber at $z = 0$. In equation,

$$P(z) = P(0)e^{-\alpha_p z}$$

is the fiber Napierian attenuation constant $\alpha_p$ in units of 1/km.

$$\alpha_p \text{ [km}^{-1}] = \frac{1}{z} \log_e \left( \frac{P(0)}{P(z)} \right)$$

Losses in dB units are additive, whereas transmission ratios are multiplicative. Thus, for a transmission distance of $z$ kilometers, the loss $\alpha z$ is in decibels (dB) and the power ratio becomes [the decadal attenuation coefficient $\alpha$ is expressed in units of dB/km].

$$\left( \frac{P(z)}{P(0)} \right) = 10^{-\alpha z/10} \approx e^{-0.230 \alpha z} \quad [\alpha \text{ in dB/km}]$$

The attenuation of an optical fiber from $z = 0$ to $L$ is given by

$$\text{Atten [dB]} = \alpha L = 10 \cdot \log_{10} \left( \frac{P(L)}{P(0)} \right)$$

Transmitted power decreases exponentially as expressed by

$$P(z) = P(0) \cdot e^{-\alpha z} \quad \text{(Lambert-Beer Law)}$$

where $\alpha$ is the fiber attenuation coefficient, and has units of $(1/\text{km})$:

$$\alpha [\text{dB/km}] = \frac{10}{z} \cdot \log_{10} \left( \frac{P(0)}{P(z)} \right) = 4.343 \alpha_p [1/\text{km}]$$

Optical Fiber Attenuation Equation (continued)

Caution:
The decadal attenuation coefficient $\alpha$ is expressed in a base-10 format,

$$\alpha \text{ [dB/km]} = \frac{10}{z} \cdot \log_{e} \left( \frac{P(z)}{P(0)} \right) = 4.343 \cdot \alpha_{p} \text{ [1/km]}$$

so we then have

$$\frac{P(z)}{P(0)} = 10^{-\alpha z/10}$$

$$\alpha z = 10 \cdot \log_{10} \left( \frac{P(z)}{P(0)} \right)$$

Note:

$$\log_{10} y = \frac{\log_{e} y}{\log_{e} 10} = \frac{\ln y}{\ln 10} = 0.4343 \times \log_{e} y \quad \text{or} \quad \log_{e} y = 2.303 \times \log_{10} y$$
# Table Relating Power Ratio to Decibels (dB)

<table>
<thead>
<tr>
<th>dB</th>
<th>power ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>10 000 000 000</td>
</tr>
<tr>
<td>90</td>
<td>1 000 000 000</td>
</tr>
<tr>
<td>80</td>
<td>100 000 000</td>
</tr>
<tr>
<td>70</td>
<td>10 000 000</td>
</tr>
<tr>
<td>60</td>
<td>1 000 000</td>
</tr>
<tr>
<td>50</td>
<td>100 000</td>
</tr>
<tr>
<td>40</td>
<td>10 000</td>
</tr>
<tr>
<td>30</td>
<td>1 000</td>
</tr>
<tr>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>3.081</td>
</tr>
<tr>
<td>3</td>
<td>1.995 (-2)</td>
</tr>
<tr>
<td>1</td>
<td>1.259</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>0.794</td>
</tr>
<tr>
<td>-3</td>
<td>0.501 (~1/2)</td>
</tr>
<tr>
<td>-6</td>
<td>0.251</td>
</tr>
<tr>
<td>-10</td>
<td>0.1</td>
</tr>
<tr>
<td>-20</td>
<td>0.01</td>
</tr>
<tr>
<td>-30</td>
<td>0.001</td>
</tr>
<tr>
<td>-40</td>
<td>0.000 1</td>
</tr>
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<td>-50</td>
<td>0.000 01</td>
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<td>0.000 000 01</td>
</tr>
<tr>
<td>-90</td>
<td>0.000 000 001</td>
</tr>
<tr>
<td>-100</td>
<td>0.000 000 000 1</td>
</tr>
</tbody>
</table>

[https://newhams.info/2017/02/23/db-or-not-db/](https://newhams.info/2017/02/23/db-or-not-db/)
The dBm

The decibel refers to ratios or relative units.

Example: When used as a ratio, assume an optical fiber with a loss of 6 dB. That means the output power is reduced by 75% compared to the fiber’s input power.

When used as an absolute power level, a common unit is optical power referenced to one milliwatt (1 mW). Example: Assume a power \( P \) and we want to express it relative to 1 mW. Then we write

\[
\text{Power level } P \text{ in dBm} = 10 \cdot \log_{10} \left( \frac{P}{P_{\text{reference}}} \right) = 10 \cdot \log_{10} \left( \frac{P}{1 \text{ mW}} \right) \text{ [dBm]}
\]

Note: If \( P_{\text{reference}} \) is one microwatt (\( \mu \text{W} \)), the unit becomes dB\( \mu \), and when referenced to one watt (1 W), the unit becomes dBW.
Consider an optical fiber of length 25 km. Find the attenuation of this fiber if the attenuation constant \( \alpha = 0.6 \text{ dB/km} \) at wavelength = 1300 nm. If the input power to the 25 kilometers of fiber is 1 milliwatt, what is the power at 25 km from the input position in the fiber?  

(Note: 1 mW \( \Rightarrow \) 0 dBm)

\[
P(0) = 1 \text{ mW at } z = 0 \\
P(z) = P(0)e^{-\alpha z}
\]

\[
P_{\text{in}} \text{ (dBm)} = 10 \cdot \log_{10} \left( \frac{P_{\text{in}} \text{ (mW)}}{1 \text{ mW}} \right) = 10 \cdot \log_{10} \left( \frac{1 \text{ mW}}{1 \text{ mW}} \right) = 0 \text{ dBm}
\]

At \( z = 25 \text{ km} \),

\[
P_{\text{out}} \text{ (dBm)} = 10 \cdot \log_{10} \left( \frac{P_{\text{out}} \text{ (dBW)}}{1 \text{ mW}} \right) = 10 \cdot \log_{10} \left( \frac{P_{\text{in}} \text{ (mW)}}{1 \text{ mW}} \right) - \alpha \times (25 \text{ km})
\]

\[
P_{\text{out}} \text{ (dBm)} = 0 \text{ dBm} - 0.6 \text{ (dB/km)} \times 25 \text{ km} = 0 \text{ dBm} - 15 \text{ dB} = -15 \text{ dBm}
\]
Next: We Review of Some Topics From Optics

What is light?
The propagation of light in vacuum & air
Rayleigh scattering
Interference
Reflection (internal & external) and
Law of Reflection
Refraction (Snell’s Law)
Huygens’ Principle
Fermat’s Principle (Principle of Least Time)
Polarization
Classical Optics

Classical Optics is divided into two main branches: Geometrical (or Ray) Optics and Physical (or Wave) Optics. In geometrical optics, light is considered to travel in straight lines, while in physical optics, light is considered as an electromagnetic wave. We will use the most convenient branch for each physical phenomena covered in EE443.

A major subfield of modern optics, Quantum Optics, deals with specifically quantum mechanical properties of light (such as with lasers).

See Section 2.2 Ray theory transmission (pages 14 to 23) and Section 2.3 Electromagnetic mode theory for optical propagation (pages 23 to 35) in Senior, 3rd edition.


Definitions of Light  https://www.merriam-webster.com/dictionary/light
1: Something that makes vision possible.
2: The sensation aroused by stimulation of the visual receptors.
3: Electromagnetic radiation of any wavelength that travels in a vacuum with a speed of 299,792,458 meters per second; specifically the range of radiation that is visible to the human eye.

What is Light?

Electromagnetic Wave

Light is electromagnetic radiation
Light is a wave
Light is energy
Light always travels at $c$

https://commons.wikimedia.org/wiki/File:EM-Wave.gif
Ray (Geometric) Optics

Ray optics, or geometric optics, is a model of optics that describes light propagation in terms of rays. The ray in geometric optics is an abstraction useful for approximating the paths along which light propagates.

Assumptions of geometrical optics are

- Propagate in straight-line paths in homogeneous mediums
- Bend at the interface between two dissimilar media
- Follow curved paths in a medium in which the refractive index changes
- May be absorbed or reflected.

However, geometrical optics does not account for certain optical effects such as diffraction and interference.
Visualizing Optical Reflection

However Optical Reflection is Surface Dependent

https://www.computerhope.com/jargon/d/diffuse-reflection.htm
Using Rotatable Mirrors to Make Optical Switches

Digression:


Lucent Technologies’ WaveStar™ Lambda Router Technology (1990)
Optical Refraction and Critical Angle

- **Critical angle** is the angle at which the refracted ray is at 90°.

https://slideplayer.com/slide/3993439/
Our Main Interest – Total internal Reflection

From Critical Angle

\[ \theta_0 = \pi / 2 \]

\[ n_1 > n_0 \]

Confined ray

This result is what allows us to make very low loss optical fibers in glass.

To Total Internal Reflection: \( \theta > \theta_C \)

No refracted ray

\[ n_1 > n_0 \]

This result is what allows us to make very low loss optical fibers in glass.
Determining the Critical Angle

Using Snell’s Law, we obtain:

\[ n_1 \cdot \sin \theta_C = n_0 \cdot \sin(90^\circ) \]

\[ \sin \theta_C = \frac{n_0}{n_1} \quad (1) \]

\[ \therefore \theta_C = \sin^{-1} \left( \frac{n_0}{n_1} \right) \]

Example: Air \((n_0 = 1.000)\) above water \((n_1 = 1.333)\)

\[ \theta_C = \sin^{-1} \left( \frac{n_0}{n_1} \right) = \sin^{-1} \left( \frac{1.000}{1.333} \right) \]

\[ \theta_C = \sin^{-1} (0.7502) = 48.61^\circ \]
Atmospheric Transmission of Electromagnetic Waves

Specific Attenuation (dB/km)

\[ \alpha \]

1 GHz

\( T = 20 \, ^\circ C; \ P = 1013 \, \text{hPa} \)

\( \text{H}_2\text{O} \) Vapor Density = 7.5 g m\(^{-3}\)

Molecular Absorption (black curve)

Free Space Optics

“Free Space Optics”

https://www.researchgate.net/figure/Specific-attenuation-across-wavelengths-from-30-cm-to-04-m-Conditions-are-for-a_fig2_235413436
Is the Speed of Light Constant?

Yes when we discuss the constancy of \( c \) we mean the speed of light in a vacuum. But, the speed of light is not constant as it moves from medium to medium. When light enters a denser medium (e.g., from air to glass) both speed and wavelength of the light wave decrease while the frequency stays the same. How much light slows down depends on the new medium's index of refraction, \( n \). (The speed of light in a medium with index \( n \) is \( c/n \).) The index of refraction is determined by the electric and magnetic properties of the medium. For air, \( n \) is 1.0003; for ice, \( n \) is 1.31; for diamond, \( n \) is 2.417.

Light always travels fastest in vacuum. Nothing can reach speeds greater than \( c \). Thus, from our equation \( v = c/n \), \( n \) must always be equal to or greater than 1. Light moves slower through denser media because more particles get in its way. Each time the light bumps into a particle of the medium, light is absorbed, causing the particle to vibrate a little, followed by the light being re-emitted. The process causes a time delay in the light's movement; so the more particles there are (i.e., the denser the medium), the more light slows, and the larger \( n \) is.

Refractive Index $n$

The refractive index is defined as the velocity of light $v$ in a dense medium relative to the velocity of light in vacuum (denoted by $c$). That is, the refractive index $n$ is defined by

$$n = \frac{c}{v} = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}$$

It is also the phase velocity in the medium. The speed of light in a vacuum is $c = 299,792,358$ meters/sec ($c$ is about $3 \times 10^8$ m/s).
## Table of Index of Refraction for Common Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Vacuum</strong></td>
<td>1</td>
</tr>
<tr>
<td><strong>Gases at 0 °C and 1 atm</strong></td>
<td></td>
</tr>
<tr>
<td>Air</td>
<td>1.000293</td>
</tr>
<tr>
<td>Helium</td>
<td>1.000036</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>1.000132</td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>1.00045</td>
</tr>
<tr>
<td><strong>Liquids at 20 °C</strong></td>
<td></td>
</tr>
<tr>
<td>Water</td>
<td>1.333</td>
</tr>
<tr>
<td>Ethanol</td>
<td>1.36</td>
</tr>
<tr>
<td>Olive oil</td>
<td>1.47</td>
</tr>
<tr>
<td><strong>Solids</strong></td>
<td></td>
</tr>
<tr>
<td>Ice</td>
<td>1.31</td>
</tr>
<tr>
<td>Fused silica (quartz)</td>
<td>1.46</td>
</tr>
<tr>
<td>PMMA (acrylic, plexiglas, lucite, perspex)</td>
<td>1.49</td>
</tr>
<tr>
<td>Window glass</td>
<td>1.52</td>
</tr>
<tr>
<td>Polycarbonate (Lexan™)</td>
<td>1.58</td>
</tr>
<tr>
<td>Flint glass (typical)</td>
<td>1.62</td>
</tr>
<tr>
<td>Sapphire</td>
<td>1.77</td>
</tr>
<tr>
<td>Cubic zirconia</td>
<td>2.15</td>
</tr>
<tr>
<td>Diamond</td>
<td>2.42</td>
</tr>
<tr>
<td>Moissanite</td>
<td>2.6</td>
</tr>
</tbody>
</table>

[https://en.wikipedia.org/wiki/Refractive_index](https://en.wikipedia.org/wiki/Refractive_index)
Rayleigh Scattering is Primary Loss Mechanism

http://hyperphysics.phy-astr.gsu.edu/hbase/atmos/blusky.html

The scattering from molecules and very tiny particles (< 1/10 wavelength) is predominantly Rayleigh scattering.

Rayleigh Scattering Explains Why the Sky Is Blue

Visualization of Rayleigh Scattering

https://www.youtube.com/watch?v=vNYef57huDE
Rayleigh Scattering in the Atmosphere

Atmospheric composition: N\textsubscript{2} (78\%), O\textsubscript{2} (21\%), Ar (1\%)

Size of N\textsubscript{2} molecule: 0.31 nm
Size of O\textsubscript{2} molecule: 0.29 nm
Size of Ar molecule: 0.3 nm
Visible wavelengths \sim 400-700 nm

Rayleigh scattering from air molecules

\[ I \alpha \frac{1}{\lambda^4} \]

The strong wavelength dependence of Rayleigh scattering enhances the short wavelengths, giving us the blue sky.

- Scattering of light off air molecules is called Rayleigh Scattering
- Involves particles much smaller than the wavelength of incident light
- Responsible for the blue color of clear sky

Dust Particles Produces a Red Sky

**Digression:**


http://www.astronomynotes.com/ismnotes/s1.htm
Destructive and Constructive Interference of Waves

https://www.tes.com/lessons/Ag5yK00uCFcDw/wave-interference
Dynamic Illustration of Interference

https://www.tes.com/lessons/Ag5yKKO0uCFCd/wave-interference
A Two-Source Interference Pattern

http://salfordacoustics.co.uk/sound-waves/superposition/interference-from-two-point-sources
Two-Source Interference Pattern

https://www.pinterest.com/pin/701013498220694093/
Fermat’s Principle (circa 1650)

Fermat’s principle states that “light travels between two points along the path that requires the least time, as compared to other nearby paths.” From Fermat’s principle, one can derive (a) the law of reflection [the angle of incidence is equal to the angle of reflection] and (b) the law of refraction [Snell’s law].

Demonstration:

http://www.surendranath.org/GPA/Optics/Fermat/Fermat.html

http://www.feynmanlectures.caltech.edu/I_26.html

http://scipp.ucsc.edu/~haber/ph5B/fermat09.pdf
Fermat’s Principle Applied to Reflection

Time $t = \left( \frac{n}{c} \right) \cdot$ distance

\[
t = \frac{n\sqrt{x^2 + a^2}}{c} + \frac{n\sqrt{(d - x)^2 + b^2}}{c}
\]

\[
\frac{dt}{dx} = 0 = \frac{nx}{2c\sqrt{x^2 + a^2}} - \frac{n(d - x)}{2c\sqrt{(d - x)^2 + b^2}}
\]

\[
\frac{x}{\sqrt{x^2 + a^2}} = \frac{(d - x)}{\sqrt{(d - x)^2 + b^2}} \Rightarrow \sin(\theta_1) = \sin(\theta_2)
\]

$\therefore \theta_1 = \theta_2$ **Law of Reflection**

\[
\sin x = \frac{\text{opposite}}{\text{hypotenuse}}
\]
Illustration of Fermat’s Principle For Refraction

The minimum time corresponds to point C, but points nearby correspond to nearly the same time (a soft minima).

http://www.feynmanlectures.caltech.edu/I_26.html
Variants of Fermat’s Principle Run Through Much of Physics

Principle of Least Action

Digression:

http://www.feynmanlectures.caltech.edu/II_19.html
Feynman’s Principle of Least Action (Field Energy)

Digression: \[ U^* = \frac{\varepsilon_0}{2} \int (\nabla \phi)^2 \, dV \]

The minimum principle says that in the case where there are conductors set at certain given potentials, the potential between them adjusts itself so that integral \( U^* \) is least. What is this integral? The term \( \nabla \phi \) is the electric field, so the integral is the electrostatic energy. The true field is the one, of all those coming from the gradient of a potential, with the minimum total energy.

Is charge evenly distributed over the outer conductor?
Law of Optical Refraction: Snell’s Law

Refractive index $n$ is the ratio of the velocity of light in a vacuum to its velocity in a specified medium. Consider the light ray P-Q.

Let $n_i = \text{index of refraction of incident medium}$

Let $n_r = \text{index of refraction of refractive medium}$

\[ n_i \cdot \sin(\theta_i) = n_r \cdot \sin(\theta_r) \]

Example:

Let $n_i = 1.00$, $n_r = 1.52$ and $\theta_i = 55^\circ$, find $\theta_r$

Snell's Law: $n_i \sin \theta_i = n_r \sin \theta_r$

\[ \sin \theta_r = \frac{n_i \sin \theta_i}{n_r} = \frac{(1.00) \sin(55^\circ)}{1.52} = 0.5389 \]

\[ \theta_r = \sin^{-1}(0.5389) = 32.6^\circ \]

https://phys.libretexts.org/Bookshelves/University_Physics/Book%3A_University_Physics_(OpenStax)/Map%3A_University_Physics_III_-_Optics_and_Modern_Physics_(OpenStax)/1%3A_The_Nature_of_Light/1.7%3A_Polarization
Why Does the Light Spectrum Spread Out From a Prism?

https://www.askiitians.com/iit-jee-ray-optics/prism/
Light Refraction Using Huygens’ Construction

http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/huygen.html
Why Light Bends in at Interfaces With Different Indices of Refraction

\[ n_1 \cdot \sin \alpha_1 = n_2 \cdot \sin \alpha_2 \]

\[ \frac{n_1}{n_2} = \frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1} \] where \( v \) is wavefront velocity

https://www.telescope-optics.net/reflection.htm
A Common Consequence of Snell’s Law

http://hyperphysics.phy-astr.gsu.edu/hbase/geoopt/gar.html
Atmospheric Refraction

https://www.timeanddate.com/astronomy/refraction.html
Diffraction Grating Spreads Electromagnetic Spectrum

Transmission Diffraction Grating Gives Multiple Spectrums

https://physics.stackexchange.com/questions/284394/second-order-spectra-vs-first-order
The angles of the diffracted modes are related to the wavelength and grating through the grating equation.

The grating equation only predicts the directions of the modes, not how much power is in them.

**Reflection Region**

\[ n_{\text{ref}} \sin \theta_m = n_{\text{inc}} \sin \theta_{\text{inc}} - m \frac{\lambda_0}{\Lambda_x} \]

**Transmission Region**

\[ n_{\text{trm}} \sin \theta_m = n_{\text{inc}} \sin \theta_{\text{inc}} - m \frac{\lambda_0}{\Lambda_x} \]

https://www.youtube.com/watch?v=OHTpxHxuAhY
Vertical Linearly Polarized Electromagnetic Wave

https://courses.lumenlearning.com/physics/chapter/27-8-polarization/
Linear,Circular & Elliptical Polarized EM Waves

http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/polclas.html
Construction of Circularly Polarized EM Wave

http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/polclas.html
Illustration of Malus’s Law

Intensity $\propto I_0^2 \cos^2(\phi)$

Malus’s Law in Action

http://olympus.magnet.fsu.edu/primer/java/polarizedlight/filters/index.html
Polarization By Double Refraction

One of refracted beams obeys Snell's law and is called Ordinary ray (O-ray). The other beam doesn't obey Snell's law and is called Extraordinary ray (E-ray).

https://www.slideshare.net/KrupeshAnadkat/polarisation-76064764
Birefringence From Double Refraction

Polarisation by double refraction:

- If beam of light is passed through certain crystal like calcite (CaCO$_3$) or quartz (SiO$_2$), it splits into two beams. These substances are called **doubly refracting** or **birefringent**.

https://www.slideshare.net/KrupeshAnadkat/polarisation-76064764
Brewster’s Angle (Polarization Angle)

**Brewster's angle** (also known as the **polarization angle**) is an angle of incidence at which light with a particular polarization is perfectly transmitted through a transparent dielectric surface, with *no reflection*. When *unpolarized* light is incident at this angle, the light that is reflected from the surface is thus perfectly polarized.

The special angle of incidence that produces a 90° angle between the reflected and refracted ray is called the Brewster angle.

https://en.wikipedia.org/wiki/Brewster%27s_angle
Laser beam has no reflection losses; only the transmitted polarized beam is traveling between the mirrors.

https://www.rp-photonics.com/brewster_windows.html

https://perg.phys.ksu.edu/vqm/laserweb/Ch-7/F7s5t1p6.htm
Equation for Brewster Angle

Brewster angle is given by: \[ \theta_p = \tan^{-1} \left( \frac{n_2}{n_1} \right) \]

If wave is incident from air to glass, \( n_1 = 1 \). Therefore,

\[ \theta_p = \tan^{-1} (n_2) \Rightarrow \tan (\theta_p) = n_2 \]
\[ \frac{\sin (\theta_p)}{\cos (\theta_p)} = n_2 \]

Applying Snell's law:
\[ \frac{\sin (\theta_p)}{\sin (\theta_r)} = n_2 \]

From both of these equations:
\[ \frac{\sin (\theta_p)}{\sin (\theta_r)} = \frac{\sin (\theta_p)}{\cos (\theta_p)} \]
\[ \Rightarrow \sin (\theta_r) = \cos (\theta_p) \]
\[ \Rightarrow \sin (\theta_r) = \sin (90^\circ - \theta_p) \]
\[ \Rightarrow \theta_r = 90^\circ - \theta_p \Rightarrow \theta_r + \theta_p = 90^\circ \]

Symbols defined on the next slide.

https://www.slideshare.net/KrupeshAnadkat/polarisation-76064764
Figure Used for Brewster Angle Computation

If a polarized wave in the plane of the paper is incident upon the glass at Brewster’s angle, then there is no reflected wave.

https://www.slideshare.net/KrupeshAnadkat/polarisation-76064764
Revisiting Optical Reflection

Using an electromagnetic field analysis a boundary at a fiber’s core-cladding interface reveals an evanescent field extending into the cladding layer which is at odds with the ray optic model.

The energy in the evanescent field is pulled back into the optical signal. This results in an effective lengthening of the optical signal path relative to the ray path interpretation.

This can be modeled by defining a “virtual reflecting plane thereby allowing the ray optical model to be retained.

https://pages.uoregon.edu/noeckel/gooshanchen/
The Goos-Hänchen effect is a phenomenon of classical optics in which a light beam reflecting off a surface is spatially shifted as if it had briefly penetrated the surface before bouncing back. The other interpretation is to add a lateral shift to the path of the optical ray path.

See Section 2.3.5 on page 35 of Senior, 3rd edition.

https://pages.uoregon.edu/noeckel/gooshanchen/
Well, That’s Enough Review of Optics

Supplementary Lecture on Diffraction

http://www.wiretechworld.com/the-future-of-optical-fibres/
What is Diffraction?

Diffraction is the bending of light when light encounters an obstacle of a size which is in the range of the wavelength of light.

Huygens-Fresnel Principle – Diffraction

Narrow Slit Case

Diffraction of a plane wave when the slit width is the size of the wavelength.

Huygens-Fresnel Principle – Diffraction

Wide Slit Case

Incoming Spherical Wave

aperture

wave front

$t_0$

$t_0 + \frac{\lambda}{c}$

$t_0 + \frac{2\lambda}{c}$

$t_0 + \frac{3\lambda}{c}$

Huygens-Fresnel Principle – Refraction

https://courses.lumenlearning.com/boundless-physics/chapter/diffraction/
Double Slit Diffraction

http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/dslit.html
Double Slit Diffraction Pattern

https://courses.lumenlearning.com/boundless-physics/chapter/diffraction/
Double Slit Diffraction Pattern

From Fresnel to Fraunhofer diffraction

Fraunhofer diffraction occurs when \( \frac{W^2}{L\lambda} \ll 1 \), where \( W \) is the aperture or slit size, \( \lambda \) is the wavelength and \( L \) is the distance from the aperture or slit.

Fraunhofer Diffraction (Far Field)

\[ \tan \theta = \frac{y}{D} \]

\[ \tan \theta \approx \sin \theta \approx \theta \approx \frac{y}{D} \]

Condition for minimum

\[ a \sin \theta = m\lambda \]

\[ y \approx \frac{m\lambda D}{a} \]

http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/fraungeo.html
Multiple Slit Diffraction – Multiple Orders

From: Eugene Hecht, Optics, 4th edition, Addison Wesley (Pearson Education Inc.) 2002, Chapter 10, Figure 19.15, page 461.
A diffraction grating is an optical component, which separates (disperses) polychromatic light into its constituent wavelengths (i.e., colors). Polychromatic light incident on the grating is dispersed so that each wavelength is reflected from the grating at a slightly different angle. The dispersion arises from the wavefront division and interference of the incident radiation from the periodic structure of the grating.

Diffraction Grating – Multiple Orders

Transmission Grating & Reflection Grating

https://www.globalspec.com/learnmore/optics_optical_components/optical_components/diffraction_gratings
Grating Equation

The angles of the diffracted modes are related to the wavelength and grating through the grating equation.

The grating equation only predicts the directions of the modes, not how much power is in them.

Reflection Region

\[ n_{\text{ref}} \sin \theta_m = n_{\text{inc}} \sin \theta_{\text{inc}} - m \frac{\lambda_0}{\Lambda_x} \]

Transmission Region

\[ n_{\text{tm}} \sin \theta_m = n_{\text{inc}} \sin \theta_{\text{inc}} - m \frac{\lambda_0}{\Lambda_x} \]

https://www.youtube.com/watch?v=OHTpxHxuAhY
Littrow-Type Grating Optical Demultiplexer

Input Fiber
