EE 443/CS 543 Optical Fiber Communications
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Fall Semester

Lecture 7

Background for Understanding Lasers

http://www.wiretechworld.com/the-future-of-optical-fibres/
Optical Sources (Optical Emitters)

The optical source is the active component in an optical fiber communication system.

Three primary categories of optical sources:
- Wideband “continuous spectra” sources (e.g., incandescent lamps)
- Monochromatic incoherent sources (e.g., light emitting diodes; LEDs)
- Monochromatic coherent sources (e.g., lasers)

Major requirements for optical fiber emitters:
- Light output is directional
- Accurately replicate the electrical input (i.e., minimize distortion and noise)
- Emit light at the wavelength of lowest fiber loss and dispersion
- Capable of signal modulation over a wide bandwidth
- Have sufficient optical power to meet system requirements
- Should have very narrow optical bandwidth (meaning laser linewidth)
- Must maintain stable operation
- Of course, it must be reliable and low cost (as always)

From: Chapter 6, Section 6.1 of Senior, 3rd ed.
Some General Comments on Optical Sources

The first generation of optical fiber communication systems operated between 800 and 900 nm using early semiconductor LED sources.

With graded index multi-mode fibers it became possible to use broad linewidth LEDs emitting in the 800 to 900 nm region. This was attractive because IR LEDs are simple, generally trouble free in their operation, and inexpensive.

When single-mode fiber was introduced the development of the narrow linewidth semiconductor laser was required. Remember with single-mode fibers the LED is not a good fit because of the difficulty in focusing sufficient light into the fiber and the wide spectrum of linewidths that accompany the LED. Recently, advanced LEDs have been developed that allow for greater power to be coupled into a fiber.

This lecture addresses the operation of the laser.

LASER stands for Light Amplification by Stimulated Emission of Radiation

From: Chapter 6, Section 6.1 of Senior, 3rd ed.
The Nature of Photons (Quanta)

Waves as particles and particles as waves! That’s quantum mechanics.

Quanta of light, called photons, must travel at the speed of light \( c = 3 \times 10^8 \) meters/sec. The fundamental relationships pertaining to photons are

\[
E = |p|c \quad \text{and} \quad E = hf = \frac{hc}{\lambda}
\]

where \( h \) is Planck’s constant \( (h = 6.626 \times 10^{-34} \text{ joule-second}) \), \( E \) is the photon energy, \( p \) is the photon’s momentum, \( f \) is the frequency of the wave and \( \lambda \) is the wavelength.

We can’t picture them in a classical way because Heisenberg’s Uncertainty Principle governs the quantum world. Waves and particle dynamics in classical physics are different realms of the World.
Atoms Possess Discrete Energy States (Example: Sodium Atom)

Absorption and Emission of Radiation

The interaction of light with matter takes place in discrete packets of energy or quanta.

Atoms possess discrete levels of energy in which light is either absorbed or emitted, causing the atom to change energy states commensurate to the two levels involved.

\[ E = E_2 - E_1 = hf , \]

where \( h \) is Planck’s constant (\( h = 6.626 \times 10^{-34} \text{ Joule-second} \)).

Photon Examples: Photoelectric effect, blackbody radiation, Bohr model of hydrogen atom, etc.
Bohr Model of the Hydrogen Atom

Niels Bohr (1913) assumed:

1. Positive charged proton and negative charged electron attract to form a stable atom
2. Stationary States correspond to orbiting electrons about the heavier proton (nucleus)
3. Orbiting electrons do not radiate energy
4. Electron transitions between different orbitals either absorb or emit photons of energy
   \[ \Delta E = E_2 - E_1 = hf \]
5. The n<sup>th</sup> orbital has a fixed angular momentum (quantized angular momentum = \( n\hbar \))
Bohr Model for Atoms Larger than Hydrogen

6 electrons
6 protons
6 neutrons

https://sites.google.com/site/mrwilkinsonphysics/classes/s1-s2-science/states-of-matter
Absorption and Emission of Radiation

From: Chapter 6, Section 6.2.1 of Senior, 3rd ed.; page 298

Initial state

Final state

\[ \Delta E = E_2 - E_1 = hf \]

(a) Absorption

(b) Spontaneous emission

(c) Stimulated emission

Random direction

Same direction

Same phase

Same polarization
The Einstein Relations

Two energy levels shown

\[ \begin{align*}
N_1 &= \frac{g_1 \cdot \exp(-E_1/kT)}{g_2 \cdot \exp(-E_2/kT)} = \frac{g_1}{g_2} \exp\left(\frac{(E_2 - E_1)/kT}{\text{Boltzmann distribution}}\right) \\
N_2 &= \frac{g_1 \cdot \exp(h\nu/kT)}{g_2} \quad \text{since} \quad E_2 - E_1 = h\nu
\end{align*} \]

where \( N_1 \) and \( N_2 \) represent the density of atoms at energy levels \( E_1 \) and \( E_2 \), respectively. Also, \( g_1 \) and \( g_2 \) represent the degeneracies of the levels, \( k \) is Boltzmann’s constant and \( T \) is absolute temperature.

The number density of atoms in state \( E_1 \) is \( N_1 \). We now want to find an expression for the rate of transitions from level 1 to level 2 is denoted by \( R_{12} \) \( \text{(i.e., absorption rate)} \)

\[ R_{12} = N_1 \rho_f B_{12} \]

where \( \rho_f \) is the spectral density of radiation and \( B_{12} \) is Einstein’s coefficient of absorption,

From: Chapter 6, Section 6.2.2 of Senior, 3rd ed.
The Einstein Relations (continued)

where \( N_1 \) and \( N_2 \) represent the density of atoms in energy levels \( E_1 \) and \( E_2 \), respectively.

Atoms in the higher state \( E_2 \) undergo transitions from level 2 to level 1 – these can be either spontaneous emissions or stimulated emissions.

The time an electron exists in the excited state \( (E_2) \) before transition \( 2 \rightarrow 1 \) is the “spontaneous lifetime” \( \tau_{21} \). Using the density of atoms in state 2 (denoted by \( N_2 \)) the spontaneous emission rate is given by

\[
R_{\text{spontaneous}} = N_2 \frac{1}{\tau_{21}} = N_2 A_{21}\quad \text{where} \quad A_{21} = \frac{1}{\tau_{21}}
\]

where \( A_{21} \) is Einstein’s coefficient of spontaneous emission. The rate of stimulated emission for the transition \( 2 \rightarrow 1 \) is

\[
R_{\text{stimulated}} = N_2 \rho_i B_{21}
\]

where \( B_{21} \) is Einstein’s coefficient of stimulated emission.

From: Chapter 6, Section 6.2.2 of Senior
The Einstein Relations (continued)

The total transition rate from level 2 to level 1 is the sum of the spontaneous rate plus the stimulated rate:

\[ R_{21} = N_2 A_{21} + N_2 \rho_f B_{21} \]

In thermal equilibrium the up (1 → 2) and down (2 → 1) transition rates are equal.

\[ R_{12} = R_{21} \]

Therefore, we can write,

\[ N_1 \rho_f B_{12} = N_2 A_{21} + N_2 \rho_f B_{21} \]

Solving for \( \rho_f \) (the spectral density of radiation) gives

\[ \rho_f = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}} = \left( \frac{A_{21}}{B_{21}} \right) \left( \frac{N_1 B_{12}}{N_2 B_{21}} \right) - 1 \]

From: Chapter 6, Section 6.2.2 of Senior, 3rd ed.
The Einstein Relations (continued)

At thermal equilibrium, the spectral density of radiation is equal to the blackbody radiation as derived by Max Planck,

\[ \rho_f = \frac{8\pi hf^3}{c^3} \left[ \frac{1}{\exp(hf/kT) - 1} \right] \]

Comparing, we note the relations

\[ B_{12} = \left( \frac{g_2}{g_1} \right) B_{21} \quad \text{and} \quad \frac{A_{21}}{B_{21}} = \frac{8\pi hf^3}{c^3} \]

If the degeneracies of the two levels are equal (let \( g_1 = g_2 \)), then \( B_{12} = B_{21} \)

\[ \frac{\text{stimulated emission rate}}{\text{spontaneous emission rate}} = \frac{B_{21}\rho_f}{A_{21}} = \frac{1}{\exp(hf/kT) - 1} \]

Next, consider Example 6.1 (on page 301 of Senior)

From: Chapter 6, Section 6.2.2 of Senior, 3rd ed.
Example 6.1 – page 301

Calculate the ratio of the stimulated emission rate to the spontaneous emission rate.

We have a lamp with an operating temperature of $T = 1000$ K. Consider the wavelength $\lambda$ to be 650 nm [not 0.5 $\mu$m (= 500 nm) as used in the textbook].

First, determine the frequency associated with a wavelength of 650 nm.

$$\text{frequency } f = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ m/sec}}{0.650 \times 10^{-6} \text{ m}} = 4.62 \times 10^{14} \text{ Hz}$$

$$\frac{B_{21}\rho_f}{A_{21}} = \frac{\text{Stimulated emission rate}}{\text{Spontaneous emission rate}} = \frac{1}{\exp\left(\frac{hf}{kT}\right) - 1}$$

$$\frac{\text{Stimulated emission rate}}{\text{Spontaneous emission rate}} = \frac{1}{\exp\left(\frac{(6.626 \times 10^{-34})(4.62 \times 10^{14})}{(1.381 \times 10^{-23})(1000)}\right) - 1}$$

$$\frac{\text{Stimulated emission rate}}{\text{Spontaneous emission rate}} = \frac{1}{\exp(22.1445) - 1} = \exp(-22.1445) = 2.414 \times 10^{-10}$$

From: Chapter 6, Section 6.2.2 (page 301) of Senior, 3rd ed.
Example 6.1 (continued)

How do we interpret the result on the last slide?

The stimulated emission event is negligible compared to the spontaneous emission of radiation. The source is incoherent. With a two-level system we can never achieve population inversion, namely

![Diagram showing Equilibrium and Nonequilibrium Population Inversion]

From: Chapter 6, Section 6.2.2 (Figure 6.2 on page 302) of Senior, 3rd ed.
Achieving Population Inversion

1. To achieve optical amplification it is necessary to create a nonequilibrium distribution of the atomic states (upper energy level must have a greater occupation count than the lower energy level). This is called population inversion.

2. Population inversion can not be created in a two-level system. Instead it requires either a three-level or a four-level system. One or more levels needs to be a “metastable” level (i.e., very slow decay).

3. “Pumping” is required to achieve population inversion. Pumping is the application of intense radiation, or an electrical current flowing across the lasing material.

From: Chapter 6, Section 6.2.3 of Senior, 3rd ed.
Population Inversion (Three-Level System)

Three-level System:

- **$E_3$**
  - Rapid decay
  - Metastable level

- **$E_2$**
  - Pumping
  - Stimulated emission

- **$E_1$**

Energy

- **$E_3$**
  - $N_3$

- **$E_2$**
  - $N_2$

- **$E_1$**
  - $N_1$

This population state can lead to laser operation.

From: Chapter 6, Section 6.2.3, Figure 6.3(a) of Senior, 3rd ed.
Population Inversion (Four-Level System)

Four-level System:

- $E_1$
- $E_2$
- $E_3$
- $E_4$

- Rapid decay
- Pumping
- Stimulated emission

Energy levels:

- $E_4$
- $E_3$
- $E_2$
- $E_1$

- $N_4$
- $N_3$
- $N_2$
- $N_1$

- 1150 nm

From: Chapter 6, Section 6.2.3, Figure 6.3(b) of Senior, 3rd ed.
Example: The Ne-He Gas Laser System

https://en.wikipedia.org/wiki/Helium%E2%80%93neon_laser
Optical Feedback and Laser Oscillation

A reflecting optical cavity provides optical feedback to form a resonator in laser devices.

We must contain the photons sufficiently long within the lasing medium and also maintain coherence. This is achieved by reflecting the beam back and forth between reflecting mirrors located at each end of the optical cavity. Multiple passes provide amplification of the radiation by repeated stimulated emission of radiation. This forms a Fabry-Pérot resonator.

The mirror at right end of the cavity is a partially transmitting reflector. That is how we extract radiation for injection into the end of an optical fiber.

From: Chapter 6, Section 6.2.4, Figure 6.3 (page 304) of Senior, 3rd ed.
Optical Cavities In General

- Plane Mirror Cavity: $R_1 = \infty$, $R_2 = \infty$
- Concentric Cavity: $R_1 = L/2$, $R_2 = L/2$
- Confocal Cavity: $R_1 = L$, $R_2 = L$
- Large Radius Cavity: $R_1 >> L$, $R_2 >> L$
- Semi-Circular Cavity: $R_1 = L$, $R_2 = \infty$
- Semi-Concave with large radius: $R_1 > L$, $R_2 = \infty$

$R_1$ = radius

https://perg.phys.ksu.edu/vqm/laserweb/Ch-8/F8s1t1p1.htm

Optical Resonator in Laser Diodes

https://circuitglobe.com/laser-diode.html
Laser Oscillation and Stable Output

A stable output is obtained under “saturation” conditions meaning the optical gain is exactly matched by the optical loss within the laser medium.

Loss mechanisms include:

1) Absorption and scattering within the amplifying medium
2) Absorption, scattering and diffraction at the cavity mirrors, and
3) Unused transmission through the mirrors (not coupled to the fiber itself)

Oscillations occur in the laser cavity over a small range of wavelengths where there is sufficient cavity optical gain to overcome the losses. Result: The laser is not perfectly monochromatic.

Broadening processes:

1) Thermal motion of atoms giving rise to Doppler shift broadening
2) Atomic collisions
3) Atomic vibrations from the thermal environment
Standing Waves in Laser Cavity

Standing waves exist only at frequencies for which the distance between the mirrors is an integer number $q$ of half wavelengths.

The resonance condition ($L$ distance between the mirrors) is given by

$$L = \frac{\lambda q}{2n}$$

where $\lambda$ is the emission wavelength and $n$ is the index of refraction of the laser medium. The emission frequency is defined by

$$f = \frac{qc}{2nL} \quad (2.13)$$

From: Chapter 6, Section 6.2.4, (pages 304-307) of Senior, 3rd ed.
The frequencies generated by $f = \frac{qc}{2nL}$ are longitudinal modes.

These modes are separated by an interval $\delta f$ calculated from

$$\delta f = \frac{c}{2nL} \quad (2.14)$$

The mode separation with respect to wavelength $\lambda$, assuming $\delta f \ll f$, and using $f = c/\lambda$, is

$$\delta \lambda = \frac{\lambda \delta f}{f} = \frac{\lambda^2}{c} \delta f \quad (2.15)$$

Therefore, mode spacing is

$$\delta \lambda = \frac{\lambda^2}{2nL} \quad (2.16)$$

This will be used to determine the laser’s linewidth as a function of $\lambda$.

From: Chapter 6, Section 6.2.4, (pages 304-307) of Senior, 3rd ed.
Example 6.2 (Page 306 in Senior) – 1

The first laser demonstrated by Theodore Maiman at Hughes Research Laboratories (announced July 7, 1960). It was a ruby laser operating at 694.3 nm and was chromium doped corundum. Suppose that the ruby cylindrical crystal with a refractive index $n = 1.78$. Assuming a wavelength of 694.3 nm (not 550 nm cited in Senior, 3rd ed.), determine the number of longitudinal modes and their frequency separation.

Corundum is extremely hard aluminum oxide, used as an abrasive. Ruby and sapphire are varieties of corundum.


https://www.semanticscholar.org/paper/Fifty-years-of-ophthalmic-laser-therapy.-Palanker-Blumenkranz/8f5c7e1d1c290d76c1a206fe373ff490e863af9/figure/0
Example 6.2 (Page 306 in Senior) – 2

Solution:
The number of longitudinal modes $q$ supported within the structure may be calculated from equation (6.13) in Senior, 3rd ed.

$$f = \frac{qc}{2nL} \Rightarrow \frac{f}{c} = \frac{1}{\lambda} = \frac{q}{2nL}$$

$$\therefore q = \frac{2nL}{\lambda} = \frac{2 \times 1.78 \times 0.04 \, \mu m}{0.6943 \times 10^{-6} \, \mu m} = 2.051 \times 10^5$$

Using equation (6.14) the frequency separation of the modes is

$$\delta f = \frac{c}{2nL} = \frac{3 \times 10^8 \, m/sec}{2 \times 1.78 \times 0.04 \, m} = 2.106 \, GHz$$

Although this is a huge number of modes, the spectral output of the laser is limited by the gain curve of the cavity size and the levels involved in the laser operation.

From: Chapter 6, Section 6.2.4, (Ex. 6.2 on page 306) of Senior, 3rd ed.
Gain Curve in Lasers (aka Gain-Bandwidth)

Gain in a laser requires population inversion. The laser gain curve depends upon the transition levels in the lasing medium involved in the population inversion between these two levels.

Analogous to Figure 6.6 (page 306) of Senior, 3rd ed.

Combining Longitudinal Modes & Laser Gain Bandwidth

The round-trip gain for the optical field within a cavity of length $L$ can be expressed as:

$$\sqrt{G_{RT}} = r_1 r_2 e^{(g - \alpha_i)L} e^{-j \frac{2 \pi n_{eff}}{\lambda} 2L}$$

Combining Longitudinal Modes & Laser Gain Bandwidth

Multi-Mode Laser Spectrum

- side-mode suppression ratio (SMSR)
- FWHM
- envelope width

Gain Curve & Longitudinal Modes With Gain Threshold Included

Multi-mode Operation

[Diagram showing relative gain vs. frequency with lasing modes and gain threshold highlighted]

https://www.semanticscholar.org/paper/Measuring-the-speed-of-light-using-beating-modes-in-D%27Orazio-Smith/c24b66221c2c8d15c95bb4400b18561bef34c7c5/figure/0
Laser Modes (a) An off-axis transverse mode is able to self-replicate after one round trip. (b) Wavefronts in self-replicating wave. (c) Four low order transverse cavity modes and Their fields. (d) Intensity patterns in the modes shown in part (c).

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https://www.networxsecurity.org/members-area/glossary/t/transverse-mode.html
Let \( \tilde{\alpha} \) (cm\(^{-1}\)) be the loss coefficient per unit length.

Then, the fractional loss can be written as

\[
\text{Fractional loss} = (r_1 r_2) \cdot \exp(-2\tilde{\alpha}L)
\]

The increase in beam intensity from stimulated emission is exponential (we simply state this and do not prove it).
Gain Threshold of Laser Cavity

Let \( \tilde{g} \) (cm\(^{-1}\)) be the gain coefficient per unit length. Then the fractional gain is given by

\[
\text{Fractional gain} = \exp(2\tilde{g}L)
\]

Hence, when the gain balances the losses, we write

\[
\exp(2\tilde{g}L) \times (r_1 r_2) \cdot \exp(-2\tilde{\alpha}L) = 1
\]

\[
(r_1 r_2) \cdot \exp[2(\tilde{g} - \tilde{\alpha})L] = 1
\]

The threshold gain per unit length is found to be

\[
\tilde{g} = \tilde{\alpha} + \frac{1}{2L} \log_e \left( \frac{1}{r_1 r_2} \right)
\]

From: Chapter 6, Section 6.2.5 of Senior, 3\(^{rd}\) ed.
Example 6.3 (page 308) in Senior

We are given a semiconductor laser diode with cavity losses of 30 cm\(^{-1}\) and a reflectivity for both mirrors (polished edges) of 30% (assume \(r_1 = r_2 = r\)). If the cavity is 600 \(\mu\)m (= 0.06 cm) long, calculate the gain coefficient per centimeter \(\tilde{g}_{th}\) needed to meet the laser’s gain threshold (gain = loss). Of course, gain must exceed the loss in a functioning laser.

Solution:

\[
\tilde{g}_{th} = \tilde{\alpha} + \frac{1}{2L} \log_e \left( \frac{1}{r r_2} \right) = \tilde{\alpha} + \frac{1}{2L} \log_e \left( \frac{1}{r} \right)^2 = \tilde{\alpha} + \frac{1}{L} \log_e \left( \frac{1}{r} \right)
\]

\[
\tilde{g}_{th} = 30 \left[ \text{cm}^{-1} \right] + \frac{1}{0.06 \left[ \text{cm} \right]} \times \log_e \left( \frac{1}{0.3} \right)
\]

\[
\tilde{g}_{th} = 30 + \frac{1.2040}{0.06} = 30 + 20.07 = 50 \left[ \text{cm}^{-1} \right]
\]
Summary: What Was Presented in This Lecture

He-Ne LASER: MODES

(a) Optical gain vs. wavelength characteristics (called the optical gain curve) of the lasing medium.
(b) Allowed modes and their wavelengths due to stationary EM waves within the optical cavity. (c) The output spectrum (relative intensity vs. wavelength) is determined by satisfying (a) and (b) simultaneously, assuming no cavity losses

Gain threshold
Three spectral lines emitted by the laser

https://slideplayer.com/slide/7452547/
Spectral Linewidth for LED and Laser Sources $\sigma_\lambda$

**From Lecture 6**

<table>
<thead>
<tr>
<th>Source</th>
<th>Linewidth (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEDs</td>
<td>20 nm to 100 nm</td>
</tr>
<tr>
<td>Semiconductor laser diodes</td>
<td>1 nm to 5 nm</td>
</tr>
<tr>
<td>Nd:YAG solid-state lasers</td>
<td>0.1 nm</td>
</tr>
<tr>
<td>NeHe gas laser</td>
<td>0.002 nm</td>
</tr>
<tr>
<td>Single Mode Laser</td>
<td>$10^{-4}$ nm</td>
</tr>
</tbody>
</table>

For an LED if center frequency is 850 nm, then a 50 nm spectral spread is 6% linewidth.
https://www.venusclubs.co.nz/02/provide-your-clients-with-answers-before-they-ask-their-questions/