Consider the following demand and cost functions:

\[ Q^D = 25 - \frac{1}{6}P \]

\[ TC = 300 + \frac{Q^2}{3} \], which represents both long run and short run costs.

(a) Derive the single price monopolistic equilibrium price and quantity. Show graphically.

Set \( MR = MC \)

To solve for \( MR \) you need to first solve for \( TR = PQ \)

From the demand curve solve for \( P = 150 - 6Q \)

\[ TR = 150Q - 6Q^2 \]

\[ MR = 150 - 12Q \]

\[ MC = \frac{2Q}{3} \]

\[ \frac{2Q}{3} = 150 - 12Q \]

\[ Q = \frac{450}{38} = 11.842 \]

\[ P = 78.95 \]

(b) Derive the profits/loss per unit and total profits/loss. Show graphically.

\[ ATC(11.842) = 29.28 \]

\[ \pi/Q = 78.95 - 29.28 = $49.67 \]

\[ \pi = 588.16 \]
(c) Does this firm exhibit economies of scale? Explain.
Yes, why?

Suppose the government wanted to regulate this monopoly.

(d) If the government imposed a $5 per unit excise tax on this monopolist, find the new equilibrium price, quantity, profits per unit and total profit. Hint: To add the per unit excise tax add 5Q to the total cost function.

\[
\begin{align*}
TC &= 300 + \frac{Q^2}{3} + 5Q \\
ATC &= \frac{300}{Q} + \frac{Q}{3} + 5 \\
MC &= 2Q/3 + 5 \\
MR &= MC \\
2Q/3 + 5 &= 150-12Q \\
\text{Solving for } Q \text{ results in } Q &= 435/38 = 11.447 \\
P &= $81.32 \\
\text{ATC}(11.447) &= 35.02 \\
\pi/Q &= 46.29 \\
\pi &= 529.93
\end{align*}
\]

(e) If the government provided a $5 per unit excise subsidy to this monopolist, find the new equilibrium price, quantity, profits per unit and total profit. Hint: To add the per unit excise subsidy subtract 5Q from the total cost function.

\[
\begin{align*}
TC &= 300 + \frac{Q^2}{3} - 5Q \\
ATC &= \frac{300}{Q} + \frac{Q}{3} - 5 \\
MC &= 2Q/3 - 5 \\
MR &= MC \\
2Q/3 - 5 &= 150-12Q \\
\text{Solving for } Q \text{ results in } Q &= 465/38 = 12.237 \\
P &= $76.58 \\
\text{ATC}(12.237) &= 23.60 \\
\pi/Q &= 52.98 \\
\pi &= 648.36
\end{align*}
\]
(f) If the government imposed a price ceiling on the monopolist, what price should the government set? Explain.

Marginal cost pricing will result in a price set where \( MC = D \)

\[
\frac{2Q}{3} = 150 - 6Q
\]

Solving for \( Q \) produces

\[
Q = 450/20 = 22.5
\]

\( P = \$15 \)

\( ATC(225) = 20.83 \)

\( \pi/Q = -5.83 \)

\( \pi = -131.24 \)

Marginal cost pricing results in a loss for natural monopolies.

Average cost pricing will result in a price where \( ATC = D \).

\[
\frac{300}{Q} + \frac{Q}{3} = 150 - 6Q
\]

Using the quadratic equation to solve for \( Q \) produces

\( Q = 21.479 \) (note there will two solutions, use the higher one)

\( P = \$21.13 \)

(g) At the ceiling price set by the government find the equilibrium quantity, profits per unit and total profit.

\( Q = 21.479 \)

\( P = \$21.13 \)

\( ATC(21.479) = 21.13 \)

\( \pi/Q = 0 \)

\( \pi = 0 \)

(h) Which of three regulatory solutions results in the best outcome for consumers?

(i) Which of three regulatory solutions results in the best outcome for the monopolist?

(j) Suppose the government allows the entrant of another firm into the market. If the entering firm has identical costs and extracts half of the market, how will this affect the equilibrium price, quantity and profits of each of the firms? Hint: To account for the decrease in demand reduce the intercept term in the demand equation by half.

From the demand curve solve for \( P = 75 - 6Q \)

\( TR = 75Q - 6Q^2 \)

\( MR = 75 - 12Q \)

\( MC = 2Q/3 \)

\( 2Q/3 = 75 - 12Q \)

\( Q = 225/38 = 5.921 \)

\( P = 39.47 \)

\( ATC(5.921) = 52.64 \)

\( \pi/Q = -13.17 \)

\( \pi = -77.96 \)

Both firms will lose money if another firm enters. In this instance, with economies of scale, the market can only sustain one firm profitably, hence firms will be reluctant to enter.
Suppose that the product represented by the demand equation above exhibits network effects.

(k) How can you modify the demand function to incorporate network effects? The consumers willingness to pay can be modified to include network effects by multiplying the original demand by Q.

\[ P(Q,Q) = 150Q - 6Q^2 \]

(l) Assume now that costs are constant at MC = $200, find the equilibrium solutions.

Set MC = D

\[ 150Q - 6Q^2 = 200 \]
\[ 6Q^2 - 150Q + 200 = 0 \]

Solve using the quadratic equation

\[ Q = 0, \]
\[ Q_L = 1.413, \]
\[ Q_H = 23.587 \]

(m) Which equilibrium solutions are stable and which are unstable? Explain.

(n) Define and explain the term critical mass. Which equilibrium represents the critical mass?