Suppose that you consume only two goods pizza \((X)\) and beer \((Y)\). Your budget on pizza and beer is $120 per month. The price of pizza is $3.00 per slice and the price of beer is $1.50 per beer.

1. What is the largest possible number of pizza slices you could consume in a month given your budget?

\[
\frac{120}{3} = 40 \text{ slices of pizza.}
\]

2. What is the largest possible number of beers you could consume in a month given your budget?

\[
\frac{120}{1.50} = 80 \text{ beers.}
\]

3. If you purchased 30 slices of pizza in a given month, how many beers could you buy?

\[
30(3) = 90. \ 120-90=30. \ \frac{30}{1.50} = 20 \text{ beers.}
\]

4. If you purchased \(X\) units of pizza in a given month, what is the formula for the number of beers \((Y)\) you could buy?

Solve the budget constrain for \(Y\).

\[
I = P_X X + P_Y Y
\]

\[
P_Y Y = I - P_X X
\]

\[
Y = \frac{I}{P_Y} - \frac{P_X}{P_Y} X
\]

5. Show graphically the budget constraint for pizza and beer. Be sure to correctly label all relevant points.

**Figure 1**
(6) Show that the slope of the budget constraint is equal to \(-\frac{P_x}{P_y}\).

Use the formula for the slope of a straight line using the X and Y intercepts as the two points i.e., \((0, \frac{I}{P_y})\) and \((\frac{I}{P_x}, 0)\).

\[
\frac{\Delta Y}{\Delta X} = \frac{Y_2 - Y_1}{X_2 - X_1} = \frac{0 - \frac{I}{P_y}}{\frac{I}{P_x} - 0} = \frac{I}{P_y} \cdot \frac{I}{P_x} = \frac{P_x}{P_y}
\]

(7) Demonstrate that at the equilibrium level of consumption, point E in Figure 2, the equilibrium condition \(\text{MRS} = \frac{P_x}{P_y}\) is equivalent to the utility maximizing rule \(\frac{\text{MU}_x}{P_x} = \frac{\text{MU}_y}{P_y}\). Explain fully.

This can easily be shown algebraically. Since \(\text{MRS} = \frac{\text{MU}_x}{\text{MU}_y}\) substitute into the equilibrium condition: \(\frac{\text{MU}_x}{\text{MU}_y} = \frac{P_x}{P_y}\). Multiply each side by \(\text{MU}_y\) and divide each side by \(P_x\) gives \(\frac{\text{MU}_x}{P_x} = \frac{\text{MU}_y}{P_y}\).

(8) Why is point A in Figure 2 not an equilibrium (i.e., utility maximizing) consumption bundle? How should income be reallocated to maximize utility? Explain fully.

At point A, \(\text{MRS} > \frac{P_x}{P_y}\), or alternatively \(\frac{\text{MU}_x}{P_x} > \frac{\text{MU}_y}{P_y}\), which means that a dollars spent on \(X\) provides greater marginal utility than \(Y\), thus the consumer should increase her consumption of \(X\) and decrease her consumption of \(Y\). As consumption of \(X\) increases its marginal utility decreases so that the ratio \(\frac{\text{MU}_x}{P_x}\) decreases. Likewise, while consumption of \(Y\) decreases its marginal utility increases so that the ratio \(\frac{\text{MU}_y}{P_y}\) increases. The consumer should keep reallocating until \(\frac{\text{MU}_x}{P_x} = \frac{\text{MU}_y}{P_y}\).
(9) Why is point B in Figure 2 not an equilibrium (i.e., utility maximizing) consumption bundle? How should income be reallocated to maximize utility? Explain fully.

At point B, \( MRS < \frac{P_x}{P_y} \), or alternatively \( \frac{MU_x}{P_x} < \frac{MU_y}{P_y} \), which means that a dollar spent on \( Y \) provides greater marginal utility than \( X \), thus the consumer should increase her consumption of \( Y \) and decrease her consumption of \( X \). As consumption of \( Y \) increases, its marginal utility decreases so that the ratio \( \frac{MU_y}{P_y} \) decreases. Likewise, while consumption of \( X \) decreases its marginal utility increases so that the ratio \( \frac{MU_x}{P_x} \) increases. The consumer should keep reallocating until \( \frac{MU_x}{P_x} = \frac{MU_y}{P_y} \).

(10) Show that the equilibrium consumption bundle is invariant to a proportional change in prices and income.

This can be demonstrated by showing that a doubling of price and income will not change the consumers choice set.

\[ 2I/2P_x = I/P_x \text{ and } 2I/2P_y = I/P_y \]

Figure 3 shows an equilibrium consumption bundle at point \( E \) when the price of \( X \) is \( P_x = \$6 \) per unit and the price of \( Y \) is \( P_y = \$3 \) per unit. Suppose the price of \( X \) falls to \$3 per unit, with equilibrium at \( E' \).

(11) What is the consumer's income at the equilibrium consumption bundle \( E \) ?

To show income, multiply price by quantity to get \( I = 6(\$3) + 2(\$6) = \$30 \). Recall that the equilibrium condition requires that income is exhausted.

(12) What is the consumer's income at the equilibrium consumption bundle \( E' \)?

Since only price has changed, income is still \$30. \( I = 5(\$3) + 5(\$3) = \$30 \).
(13) Identify the substitution effect of the price change.

The substitution effect is the movement from 2 to 4 units of good x along the original indifference curve U₁.

(14) Identify the income effect of the price change. Is good X a normal good or an inferior good?

The income effect is the shift from 4 to 5 units of good X illustrating that good X is a normal good.

(15) Derive the equation for a linear demand curve for good X from the information in Figure 3. Show graphically.

\[ P/\text{unit} \]

\[ \begin{array}{c|c|c}
P & Q & \text{Demand} \\
\hline
$6 & 2 & \text{Quantity} \\
$3 & 5 & \\
\end{array} \]

Given the two points, derive the equation for a linear demand \( Q^D = a - bP \)

Use the formula for the slope of a straight line using the X and Y intercepts as the two points i.e., \( P_1Q_1 = (3,5) \) and \( P_2Q_2 = (6,2) \).

\[
b = \frac{\Delta Q}{\Delta P} = \frac{Q_2 - Q_1}{P_2 - P_1} = \frac{2 - 5}{6 - 3} = \frac{-3}{3} = 1 \]

\[ a = 5 + 1(3) = 8 \]

\[ Q^D = 8 - P \]
(16) Suppose a consumer’s marginal rate of substitution of Y for X is 5 (that is $MU_x / MU_y = 5$) the price of X is $9.00 per unit and the price of Y is $2.00 per unit. Is this consumer spending too much of her income on Y. Explain your answer and show graphically.

Using the utility maximizing condition, you can see that $MU_x / MU_y = 5 > P_x / P_y = 9/2$, which indicates that this consumer spending too much of her income on Y. To increase her utility she should spend more on X and less on Y.

(17) Suppose that a rational consumer consumes only two goods X and Y. Assume that her marginal rate of substitution of Y for X is given by the following formula:

$$MRS = MU_x / MU_y = Y/X$$

That is the consumers MRS is simply equal to the ratio of the number of units of Y consumed to the number of units of X consumed. Assume that the consumers income is $100, the price of X is $5 per unit and the price of Y is $10 per unit. What is the equilibrium quantity of X and Y consumed?

HINT: Use the equilibrium conditions to solve the problem i.e.,
1. $MRS = P_x / P_y$
2. $I = P_x X + P_y Y$

From the first equilibrium condition we know that $Y/X = 5/10$ or that $Y = (1/2)X$.
Substitute this into the second equilibrium condition and solve for X:
$100 = 5X + 10((1/2)X)$ => $X = 10$ units.
Substituting this into $Y = (1/2)X$ => $Y = 5$ units.
The utility maximizing bundle 10 units of X and 5 units of Y.