3.5 (i) No. By definition, \( \text{study} + \text{sleep} + \text{work} + \text{leisure} = 168 \). Therefore, if we change \text{study}, we must change at least one of the other categories so that the sum is still 168.

(ii) From part (i), we can write, say, \text{study} is a perfect linear function of the other independent variables: \( \text{study} = 168 - \text{sleep} - \text{work} - \text{leisure} \). This holds for every observation, so MLR.3 violated.

(iii) Simply drop one of the independent variables, say \text{leisure}:

\[
\text{GPA} = \beta_0 + \beta_1 \text{study} + \beta_2 \text{sleep} + \beta_3 \text{work} + u.
\]

Now, for example, \( \beta_1 \) is interpreted as the change in GPA when \text{study} increases by one hour, where \text{sleep}, \text{work}, and \( u \) are all held fixed. If we are holding \text{sleep} and \text{work} fixed but increasing \text{study} by one hour, then we must be reducing \text{leisure} by one hour. The other slope parameters have a similar interpretation.

3.10 (i) Because \( x_1 \) is highly correlated with \( x_2 \) and \( x_3 \), and these latter variables have large partial effects on \( y \), the simple and multiple regression coefficients on \( x_1 \) can differ by large amounts. We have not done this case explicitly, but given equation (3.46) and the discussion with a single omitted variable, the intuition is pretty straightforward.

(ii) Here we would expect \( \hat{\beta}_1 \) and \( \hat{\beta}_1 \) to be similar (subject, of course, to what we mean by “almost uncorrelated”). The amount of correlation between \( x_2 \) and \( x_3 \) does not directly affect the multiple regression estimate on \( x_1 \) if \( x_1 \) is essentially uncorrelated with \( x_2 \) and \( x_3 \).

(iii) In this case we are (unnecessarily) introducing multicollinearity into the regression: \( x_2 \) and \( x_3 \) have small partial effects on \( y \) and yet \( x_2 \) and \( x_3 \) are highly correlated with \( x_1 \). Adding \( x_2 \) and \( x_3 \) like increases the standard error of the coefficient on \( x_1 \) substantially, so \( \text{se}(\hat{\beta}_1) \) is likely to be much larger than \( \text{se}(\hat{\beta}_1) \).

(iv) In this case, adding \( x_2 \) and \( x_3 \) will decrease the residual variance without causing much collinearity (because \( x_1 \) is almost uncorrelated with \( x_2 \) and \( x_3 \), so we should see \( \text{se}(\hat{\beta}_1) \) smaller than \( \text{se}(\hat{\beta}_1) \). The amount of correlation between \( x_2 \) and \( x_3 \) does not directly affect \( \text{se}(\hat{\beta}_1) \).
3.12 (i) The shares, by definition, add to one. If we do not omit one of the shares then the equation would suffer from perfect multicollinearity. The parameters would not have a ceteris paribus interpretation, as it is impossible to change one share while holding all of the other shares fixed.

(ii) Because each share is a proportion (and can be at most one, when all other shares are zero), it makes little sense to increase $share_p$ by one unit. If $share_p$ increases by .01 – which is equivalent to a one percentage point increase in the share of property taxes in total revenue – holding $share_I, share_S$, and the other factors fixed, then growth increases by $\beta_1 (.01)$. With the other shares fixed, the excluded share, $share_F$, must fall by .01 when $share_p$ increases by .01.

C3.1 (i) Probably $\beta_2 > 0$, as more income typically means better nutrition for the mother and better prenatal care.

(ii) On the one hand, an increase in income generally increases the consumption of a good, and $cigs$ and $faminc$ could be positively correlated. On the other, family incomes are also higher for families with more education, and more education and cigarette smoking tend to be negatively correlated. The sample correlation between $cigs$ and $faminc$ is about $- .173$, indicating a negative correlation.

(iii) The regressions without and with $faminc$ are

$$\bar{bwght} = 119.77 - .514 \ cigs$$

$n = 1,388, \ R^2 = .023$

and

$$\bar{bwght} = 116.97 - .463 \ cigs + .093 \ faminc$$

$n = 1,388, \ R^2 = .030$. 

The effect of cigarette smoking is slightly smaller when $faminc$ is added to the regression, but the difference is not great. This is due to the fact that $cigs$ and $faminc$ are not very correlated, and the coefficient on $faminc$ is practically small. (The variable $faminc$ is measured in thousands, so $10,000$ more in 1988 income increases predicted birth weight by only .93 ounces.)
(i) The constant elasticity equation is

\[
\log(salary) = 4.62 + 0.162 \log(sales) + 0.107 \log(mktval)
\]

\[n = 177, \quad R^2 = .299.\]

(ii) We cannot include profits in logarithmic form because profits are negative for nine of the companies in the sample. When we add it in levels form we get

\[
\log(salary) = 4.69 + 0.161 \log(sales) + 0.098 \log(mktval) + 0.00036 \text{ profits}
\]

\[n = 177, \quad R^2 = .299.\]

The coefficient on profits is very small. Here, profits are measured in millions, so if profits increase by $1\text{ billion}$, which means $\Delta \text{profits} = 1,000$ – a huge change – predicted salary increases by about only 3.6%. However, remember that we are holding sales and market value fixed.

Together, these variables (and we could drop profits without losing anything) explain almost 30% of the sample variation in $\log(salary)$. This is certainly not “most” of the variation.

(iii) Adding ceoten to the equation gives

\[
\log(salary) = 4.56 + 0.162 \log(sales) + 0.102 \log(mktval) + 0.00029 \text{ profits} + 0.012 \text{ ceoten}
\]

\[n = 177, \quad R^2 = .318.\]

This means that one more year as CEO increases predicted salary by about 1.2%.

(iv) The sample correlation between $\log(mktval)$ and profits is about .78, which is fairly high. As we know, this causes no bias in the OLS estimators, although it can cause their variances to be large. Given the fairly substantial correlation between market value and firm profits, it is not too surprising that the latter adds nothing to explaining CEO salaries. Also, profits is a short term measure of how the firm is doing while mktval is based on past, current, and expected future profitability.
3.8 (i) The average of prpblick is .113 with standard deviation .182; the average of income is 47,053.78 with standard deviation 13,179.29. It is evident that prpblick is a proportion and that income is measured in dollars.

(ii) The results from the OLS regression are

\[
\sqrt{psoda} = .956 + .115 \text{ prpblick} + .000016 \text{ income}
\]

\[
n = 401, \quad R^2 = .064.
\]

If, say, prpblick increases by .10 (ten percentage points), the price of soda is estimated to increase by .0115 dollars, or about 1.2 cents. While this does not seem large, there are communities with no black population and others that are almost all black, in which case the difference in psoda is estimated to be almost 11.5 cents.

(iii) The simple regression estimate on prpblick is .065, so the simple regression estimate is actually lower. This is because prpblick and income are negatively correlated (-.43) and income has a positive coefficient in the multiple regression.

(iv) To get a constant elasticity, income should be in logarithmic form. I estimate the constant elasticity model:

\[
\log(psoda) = -.794 + .122 \text{ prpblick} + .077 \log(income)
\]

\[
n = 401, \quad R^2 = .068.
\]

If prpblick increases by .20, \(\log(psoda)\) is estimated to increase by .20(.122) = .0244, or about 2.44 percent.

(v) \(\hat{\beta}_{prpblick}\) falls to about .073 when prppov is added to the regression.

(vi) The correlation is about -.84, which makes sense because poverty rates are determined by income (but not directly in terms of median income).

(vii) There is no argument that they are highly correlated, but we are using them simply as controls to determine if the is price discrimination against blacks. In order to isolate the pure discrimination effect, we need to control for as many measures of income as we can; including both variables makes sense.