Sex, Drugs and the Alchian-Allen Theorem

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Abstract

A recent series of studies have observed that women taking the contraceptive pill prefer more masculine men than women not on the pill. Each discipline that has studied this phenomena has come up with their own hypothesis: Psychologist have attributed the phenomena to a unconscious psychological reaction to the pill, the medical profession has attributed it to a biological change in preferences towards more masculine men, and evolutionary psychologists have attributed it to change in olfactory acuteness. To this we add an economic explanation, where women on the pill react to a change in the relative cost of sexual intimacy. In this paper we show that the observed behavior of women on the pill is fully explained by what has been called the "third law of demand" or the Alchian-Allen Theorem. In its most common form, the Alchian-Allen Theorem is often associated with the phenomena in which a common fixed transportation or shipping cost added to the price of a high and low quality good results in a relative increase in consumption of the high quality good. This paper generalizes the Alchian-Allen Theorem and treats the contraceptive pill as a common fixed cost applied to the relative "price" of sexual activity with masculine or sensitive men. Given this choice structure, we find that the inclusion of the contraceptive pill would lead rational women to choose more masculine men as sexual partners.

THE ALCHIAN-ALLEN THEOREM

The Alchian-Allen Theorem is often associated with the phenomena in which a common fixed transportation or shipping cost added to the price of a high and low quality good results in a relative increase in consumption of the high quality good. To illustrate this concept, Borcherding and Silberberg (1978) describe a letter to the *Seattle Times* in which an irate local consumer found it difficult to find quality apples in Seattle, Washington, a state known for its apples. The authors, now famous answer to this apparent contradiction was an application of the Alchian-Allen Theorem. The following brief excerpt illustrates the theorem.[1]

> Suppose, for example, a 'good' apple costs 10 cents and a 'poor' apple 5 cents locally. Then, since the decision to eat on good apple costs the same as eating two poor apples, we can say that a good apple in essence costs two poor apples. Two good apples cost four poor apples.
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> Suppose now that it costs 5 cents per apple (any apple) to ship apples East. Then, in the East, good apples will cost 15 cents and poor ones 10 cents each. But now eating two good apples will cost three-not four poor apples.
>
> Though both prices are higher, good apples have become relatively cheaper, and a higher percentage of good apples will consumed in the East than here.

Because the common fixed cost lowers the relative price of the high quality apples, a greater proportion of high quality apples will be consumed in distant locations. As a result, apple suppliers will ship high quality apple to the distant locations. As a result of the 1978 paper by Borcherding and Silberberg, the theorem is often called the "shipping the good apples out" theorem. The result can be shown graphically. Assume that local apple consumers face the budget constraint represented by line segment I/P_P and I/P_G, with preferences represented by U_{local}. The utility maximizing consumption of poor and good apples is shown as $Good_{Local}$ and $Poor_{Local}$. Now, facing the common transportation cost "t",

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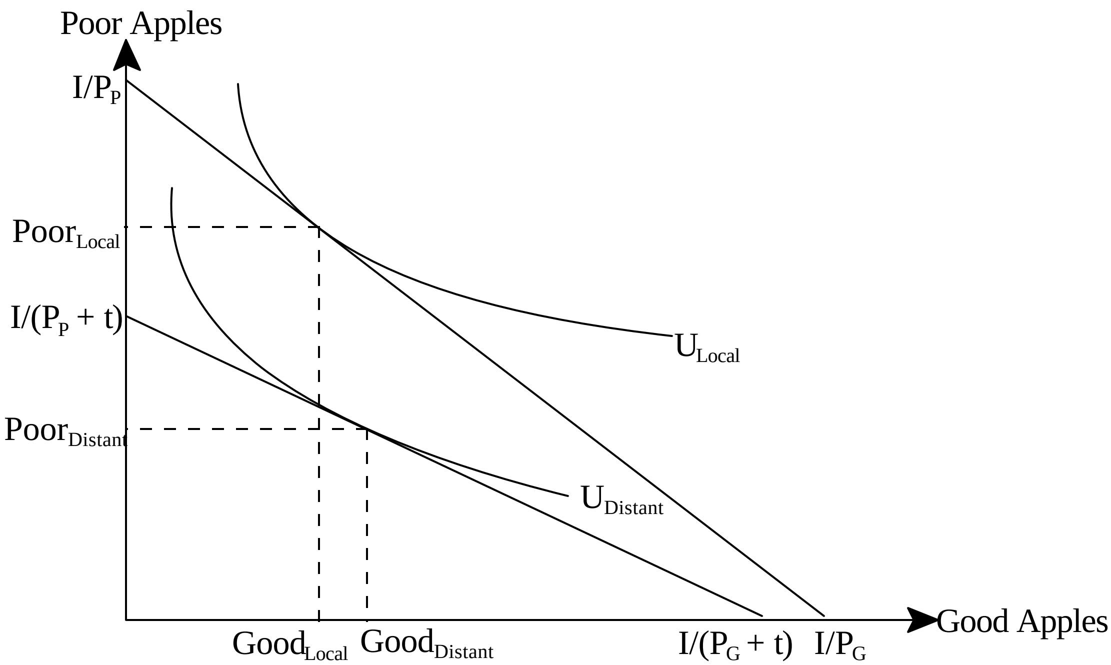
[1] Borcherding and Silberberg (1978).



distant consumers face the budget constraint represented by line segment I/(P_P + t) and I/(P_G +t). Because the common shipping costs raises the cost of both types of apples in distant locations, the budget constraint for distant consumer is below that of local consumers. However, the transportation cost raises the cost of both types of apples, it results in a disproportionate price increase of the poor quality apples resulting in a lower relative price of good apples and a flatter budget constraint for distant consumers (i.e., (P_G +t) > P_G and (P_P + t) > P_P but (P_G + t)/(P_P +t) < P_G /P_P). The transportation costs result in distant consumers consuming less apples than local consumers because prices are higher, however since relative price changed, distant consumers will consume relatively more good apples than local consumers. The theorem essentially formalizes the fact that relative prices matter in consumption choices. Borcherding and Silberberg go on to show that it does not matter whether the goods are shipped to the consumers or the consumers are shipped to the goods and that the theorem is applicable to common fixed costs other than shipping. Empirical support of the theorem is supplied by Bertonazzi, Maloney and McMormick (1993), who find that those traveling to football games purchase more expensive seats than local residents.

Formally, the Alchian-Allen Theorem can be shown to hold for the general case in which consumers maximize utility across n-goods denoted $x_1, ..., x_n$. Optimization of well behaved preferences result in Hicksian compensated demand functions $x_i^U(p_1, ..., p_n, U)$.[2] We can define the first two goods as two versions of the same good, so that x_1 is the high quality good and x_2 is the low quality good. If we assume that price increases with quality, then $p_1 > p_2$. Borcherding and Silberberg (1978) express the Alchian-Allen theorem as,

$$\frac{\partial(x_1^U/x_2^U)}{\partial t} > 0, \tag{1}$$

where t is a common fixed cost added to the price of both x_1 and x_2. Dropping the superscripts, and combining the facts that,

$$\frac{\partial x_i}{\partial t} = \frac{\partial x_i}{\partial p_1} + \frac{\partial x_i}{\partial p_2} \text{ and } \frac{\partial(x_1/x_2)}{\partial t} = \frac{1}{x_2^2}\left(\frac{\partial x_1}{\partial t}x_2 - x_1\frac{\partial x_2}{\partial t}\right).$$

The Alchian-Allen theorem can be restated as,

$$\frac{\partial(x_1/x_2)}{\partial t} = \frac{1}{x_2^2}\left[\left(\frac{\partial x_1}{\partial p_1} + \frac{\partial x_1}{\partial p_2}\right)x_2 - x_1\left(\frac{\partial x_2}{\partial p_1} + \frac{\partial x_2}{\partial p_2}\right)\right]. \tag{2}$$

Moving x_1 and x_2 outside the square brackets and multiplying each term inside the square brackets by

[2] The utility functions have the usual conditions of being monotonic, continuously twice differentiable and strictly quasi-concave.

one produces,

$$\frac{\partial(x_1/x_2)}{\partial t} = \frac{x_1}{x_2}\left[\left(\frac{\partial x_1}{\partial p_1}\frac{p_1}{x_1}\frac{1}{p_1} + \frac{\partial x_1}{\partial p_2}\frac{p_2}{x_2}\frac{1}{p_2}\right) - \left(\frac{\partial x_2}{\partial p_1}\frac{p_1}{x_2}\frac{1}{p_1} + \frac{\partial x_2}{\partial p_2}\frac{p_2}{x_2}\frac{1}{p_2}\right)\right] \tag{3}$$

which can be rewritten as,

$$\frac{\partial(x_1/x_2)}{\partial t} = \frac{x_1}{x_2}\left[\frac{e_{11}}{p_1} + \frac{e_{12}}{p_2} - \frac{e_{21}}{p_1} - \frac{e_{22}}{p_2}\right]. \tag{4}$$

Homogeneity of the compensated demand functions and application of Euler's theorem results in the fact,[3]

$$\sum_{j}^{n} e_{ij} = 0, \ \forall\, i. \tag{5}$$

Consider first the case of a two good world where x_1 and x_2 are forced to be substitutes. In this case $e_{ij} = -e_{ji}$. Substituting this into the equation above we get the result,

$$\frac{\partial(x_1/x_2)}{\partial t} = \frac{x_1}{x_2}\left[(e_{11} - e_{21})\left(\frac{1}{p_1} - \frac{1}{p_2}\right)\right]. \tag{6}$$

x_1/x_2 is clearly positive, so we can examine the terms in the square brackets. The first term in the first set of parentheses is negative since compensated own price elasticities are negative ($e_{ii} < 0$) while the second term, the cross price effect is positive ($e_{ij} > 0$) by definition. The second term is negative by the initial assumption that $p_1 > p_2$ so that $1/p_1 < 1/p_2$. Thus we get the result that,

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[3] This result is often referred to as Hicks's third law.

$$\frac{\partial(x_1^U/x_2^U)}{\partial t} > 0$$

which is the Alchian-Allen theorem.

In an n-good world we can substitute the more general form of Hick's third law into equation (4) to get

$$\frac{\partial(x_1/x_2)}{\partial t} = \frac{x_2}{x_2}\left[\frac{\epsilon_{11}}{p_1} + \frac{-\sum_{j=1}^{n}\epsilon_{1j}}{p_2} - \frac{\epsilon_{21}}{p_1} - \frac{-\sum_{j=1}^{n}\epsilon_{2j}}{p_2}\right] \tag{8}$$

which can again be rewritten as,

$$\frac{\partial(x_1/x_2)}{\partial t} = \frac{x_1}{x_2}\left[(\epsilon_{11}-\epsilon_{21})\left(\frac{1}{p_1}-\frac{1}{p_2}\right)\right]+\frac{1}{p_2}\sum_{j\geq 3}^{n}(\epsilon_{2j}-\epsilon_{1j}) \tag{9}$$

At first glance, equation (9) may appear very similar to equation (6) with the addition of second half of the equation. There is however, a very important difference. In equation (6), the two good model, the goods are substitutes by definition. In equation (9), this restriction is not imposed. That is, $\epsilon_{21} > 0$ need not be positive. What is need to satisfy Alchian -Allen is not necessarily that goods one and two be substitutes but that the two good not be highly complementary i.e., $\epsilon_{21} > \epsilon_{11}$.[4] For the most part however, the theorem applies to goods that are viewed by consumers to be substitutes. In this case, the first part of equation (9) will be positive. Moreover, as the goods become closer substitutes,

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[4] See Bauman (2004) for an elaboration of this point.

Borcherding and Silberberg show that the first part of equation will become larger while the second part will tend toward zero, thus maintaining the Alchian-Allen theorem.[5]

A more generalized statement of the Alchian-Allen Theorem is that any common cost added to or subtracted from the price of goods that results in a relative price change of the goods will result in a change in the relative consumption of those goods. In the next section, we set up a model in which contraceptive action lowers the cost of sexual intimacy by a common amount but results in a disproportionate effect on the price of intimacy with men of differing costs of intimacy. The predictable response of a change in the relative preference is shown to be the result of the change in relative price.

SEX AND DRUGS

The Alchian-Allen theorem has been applied to a variety of situations in which the relative consumption of high and low quality commodities vary directly by cost. The recent literature on mate selection and the pill has focused on the selection between "masculine" men and "sensitive" men. In this paper, we develop a model that analyze mate selection between two alternative choices ("masculine" and "sensitive" men) that vary by cost. Our notion of costs associated with sexual intimacy include the probability of getting pregnant and we assume that this probability is common to both masculine and sensitive men. However, if a woman does become pregnant, there is also the relationship costs associated with childbearing. These include the emotional and financial cost associated with raising a child. For example, if sensitive men are more likely to make trustworthy and faithful partners, then the cost of sexual intimacy and potential childbirth with these men can be thought of a being relatively low.

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[5] Borcherding and Silberberg (1978).

Conversely, if the more masculine men are less likely to make good partners after intimacy and childbirth, then the cost of sexual intimacy with them is relatively high.

The effect of contraceptive action, for example the contraceptive pill, will be to reduce the probability of sexual activity resulting in childbirth. We assume that the contraceptive pill will reduce the probability of pregnancy by a common amount for both masculine and sensitive men. However, just as adding a common transportation cost to high and low cost goods will result in a greater proportional change in the low cost good. Reducing the probability of pregnancy by a common amount will result in a disproportionate effect on the cost of sexual activity with a high relationship cost partner.

We can model this as follows. Suppose that women choose between only two types of partners, masculine types denoted X^M and sensitive types denoted X^S. Let B^M and B^S denote the utility derived from intimacy with each type respectively and assume that $B^M > B^S$.

The cost of intimacy associated with each partner can be represented by the following,

$$p(?) \cdot \sum_t^n \frac{C_t^M}{(1+r)^t} \quad \text{and}$$

$$p(?) \cdot \sum_t^n \frac{C_t^S}{(1+r)^t},$$

where p(?) is the probability of getting pregnant, which is common to both cost functions. Assume also that $C^M > C^S$, which says that the cost of sexual activity resulting in pregnancy is greater for masculine men than for sensitive men. The cost function C^i can be thought of as consisting of both emotional and financial costs. Thus,

$C^i = E^i + F^i$ for i =M and S.

Where E^i is the emotional cost associated with childbearing with partner i and F^i is the financial cost associated with childbearing with partner i. We can assume that $E^M > E^S$, the emotional cost of childbearing is greater with masculine men, if for example they are less caring than sensitive men. We can also assume that $F^M > F^S$, the financial cost is greater for masculine men if for example sensitive men are better providers and may remain in relationships longer than masculine men. The net cost of sexual intimacy facing women is then,

$$P^M = B^M - p(?) \cdot \sum_t^n \frac{C_t^M}{(1+r)^t}$$

$$P^S = B^S - p(?) \cdot \sum_t^n \frac{C_t^S}{(1+r)^t}.$$

Women then face a relative cost of sexual intimacy with a masculine man of P^M/P^S. Assuming rational consumption behavior, women face a compensated demand function for each type of sexual partner of the form,

$$x_M^U(P^M, P^S, U) \text{ and } x_S^U(P^M, P^S, U)$$

for masculine and sensitive men respectively. From the cost function, it can be seen that as long as $B^M/B^S > C^M/C^S$, then contraceptive action reduces the net cost of sexual intimacy by more for masculine men than it does for sensitive men and $\partial(P^M/P^S)/\partial p(?) < 0$. That is, a decrease in the probability of getting pregnant reduces the cost of sexual intimacy with masculine men by more than it will reduce the cost sexual intimacy with sensitive men. The lower relative cost thus induces the predictable result that women on the pill choose more masculine men over sensitive men (i.e., $M(X_M /X_S)/Mp(?) < 0$). Although the sign is reversed, we get results analogous to those of "shipping the good apples out" where the inclusion of a common cost results in disproportionate effects. As such, we should observe analogous results with mate selection. Contraceptive action not only lowers the cost of sexual intimacy, it lowers the relative cost of sexual intimacy with masculine men. We should thus observe two phenomena. First, we should observe women on the pill engaging in more sex than women not taking the pill. Second, we should observe women on the pill choosing more masculine sexual partners than women not on the pill.

It is the second observation that is consistent with the behavior observed in the recent studies of mate selection.

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