Binary Phase Shift Keying (BPSK)

- In BPSK, the symbol mapping table encodes bits \((b_n)\) 1 and 0 to transmission symbols \((a_n)\) 1 and –1, respectively.

- Every \(T_b\) seconds the modulator transmits one of the two carrier bursts that corresponds to the information bit being a 1 or 0.

Binary 1: \(s_1(t) = A_c \cos(2\pi f_c t), \quad 0 \leq t \leq T_b\)

Binary 0: \(s_2(t) = A_c \cos(2\pi f_c t + \pi) = -A_c \cos(2\pi f_c t)\)

- The resultant BPSK signal can be expressed as

\[
x(t) = A_c \sum_{n=-\infty}^{\infty} a_n \Pi \left[ \left( t - nT_b \right) / T_b \right] \cos \left( 2\pi f_c t \right), \quad a_n \in \mathcal{H} = \{1, -1\}
\]

- \(x(t)\) contains only the in-phase component \(I(t)\); \(Q(t)\) is zero.
BPSK Modulation

\[ I(t) = \sum_{n=-\infty}^{\infty} a_n \Pi \left[ (t-nT_b)/T_b \right] \]

\[ x(t) = A_c \sum_{n=-\infty}^{\infty} a_n \Pi \left[ (t-nT_b)/T_b \right] \cos(2\pi f_c t) \]

<table>
<thead>
<tr>
<th>Symbol mapping</th>
<th>$b_n$</th>
<th>$a_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
<td></td>
</tr>
</tbody>
</table>

SM = Symbol mapping
PSF = Pulse shaping filter

Information $b_n$

Baseband signal

\[ I(t) = \sum_{n=-\infty}^{\infty} a_n \Pi[(t-nT_b)/T_b] \]

Modulated signal

\[ x(t) = A_c \sum_{n=-\infty}^{\infty} a_n \Pi[(t-nT_b)/T_b] \cos(2\pi f_c t) \]
BPSK Coherent Demodulation

\[ r(t) = x(t) + n(t) \]

BPSK demodulator

Threshold comparator

SM = Symbol mapping

After multiplication at receiver
\[ x(t) \times 2\cos(2\pi f_c t) \]

Baseband signal after LP filtering

Recovered information \( b_n \)
Error Performance

- If we choose the basis function
  \[ \phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t), \quad 0 \leq t \leq T_b \]
  we can write BPSK waveforms as
  \[ s_1(t) = A_c \sqrt{\frac{T_b}{2}} \phi_1(t) = \sqrt{E_b} \phi_1(t) \]
  \[ s_2(t) = -A_c \sqrt{\frac{T_b}{2}} \phi_1(t) = -\sqrt{E_b} \phi_1(t) \]

- BPSK is thus polar signaling with \( d_{\text{min}} = 2\sqrt{E_b} \)

- The BER performance of BPSK is, therefore, identical to that of polar NRZ signaling
  \[ \text{BER}_{\text{BPSK}} = Q\left( \frac{d_{\text{min}}}{\sqrt{2N_o}} \right) = Q\left( \sqrt{\frac{2E_b}{N_o}} \right) \]
Binary Frequency Shift Keying (BFSK)

- In BFSK, information is transmitted by sending carrier bursts of two different frequencies, \( f_1 = f_c + \Delta f / 2 \) and \( f_2 = f_c - \Delta f / 2 \), to transmit binary data. \( \Delta f \) is called the *frequency deviation*

\[
\text{Binary 1: } s_1(t) = A_c \cos(2\pi f_c t + \pi \Delta f t + \phi_1), \quad 0 \leq t \leq T_b
\]

\[
\text{Binary 0: } s_2(t) = A_c \cos(2\pi f_c t - \pi \Delta f t + \phi_2), \quad 0 \leq t \leq T_b
\]

- A simple way to generate a BFSK signal is to use two separate oscillators tuned to frequencies \( f_1 \) and \( f_2 \) and switch between their outputs in accordance with the amplitude of the random data bit during that bit interval

- \( \phi_1 \) and \( \phi_2 \) are arbitrary phases of two frequency bursts generated by separate oscillators
Other Demodulation Techniques

- Coherent demodulation may neither be desirable nor feasible in many practical applications.
  - The propagation delay on some radio channels changes too rapidly to permit accurate tracking of the carrier phase at the demodulator
  - Tracking the incoming signal’s carrier phase and synchronizing the demodulator to it requires additional hardware complexity with cost and power efficiency ramifications
- *Differentially Coherent Demodulator* – demodulator uses the carrier phase of the previous symbol period as phase reference for the current period
- *Noncoherent Demodulator* – demodulator does not exploit phase information in the received signal for its demodulation
DBPSK (contd)

**DBPSK modulator**

\[ I(t) = \sum_{n=-\infty}^{\infty} a_n \Pi \left( \frac{t-nT_b}{T_b} \right) \]

\[ x(t) = \sqrt{\frac{2E_b}{T_b}} \sum_{n=-\infty}^{\infty} a_n \Pi \left( \frac{t-nT_b}{T_b} \right) \cos(2\pi f_c t) \]

<table>
<thead>
<tr>
<th>Symbol mapping</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_n )</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>

PSF = Pulse shaping filter

**DBPSK demodulator**

\[ r(t) = x(t) + n_f(t) \]

Threshold comparator

SM = Symbol mapping
DBPSK (contd)

- The output of the sampler is given by

\[ r_o = \begin{cases} 
E_b + n(T_b), & a_n = a_{n-1} \\
-E_b + n(T_b), & a_n \neq a_{n-1}
\end{cases} \]

where \( n(t) \) is non-Gaussian noise.

- Since we have polar symmetry, \( V_T = 0 \) is selected. We can now write the following decision rule for decoding

\[ r_o > 0 \Rightarrow \hat{a}_n = \hat{a}_{n-1} \Rightarrow \hat{b}_n = 0 \]

\[ r_o < 0 \Rightarrow \hat{a}_n \neq \hat{a}_{n-1} \Rightarrow \hat{b}_n = 1 \]

- The probability of bit error for DBPSK scheme is given by

\[ BER_{DBPSK} = \frac{1}{2} e^{-\frac{E_b}{N_o}} \]
DBPSK (contd)

Table 11.2 Example of Differential Decoding of BPSK

<table>
<thead>
<tr>
<th>Differentially encoded bits $d_n$</th>
<th>1</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>1</th>
<th>0</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold-comparison sign</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Decoded differential bits $\hat{d}_n$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Regenerated data bits $\hat{b}_n$</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

![Graph showing BER vs. $E_b/N_0$ for BPSK and DBPSK]
BER Comparison

Probability of Bit Error Rate vs. (Eb/No)dB

- BPSK
- DBPSK
- Coherent ASK/FSK
- Noncoherent ASK/FSK
Quadrature Modulation Schemes

- In BPSK the phase of the carrier burst is shifted 0 or 180 degrees every pulse or symbol interval depending upon the information sequence. Thus each modulated carrier pulse transmits 1 bit of information.

- If, on the other hand, the modulation scheme can use phase shifts of 45, 135, 225, or 315 degrees, each modulated carrier pulse transmits 2 bits of information. This technique is called Quadrature Phase Shift Keying (QPSK).
  - Using QPSK, we can *double* the data rate over the same channel bandwidth.

- QPSK is one of the modulation methods in the family known as Quadrature modulation schemes which are widely used, including in cellular and cable modem applications.
Quadrature Modulation Schemes (contd)

- Suppose an information source generates $M$-ary symbols at a rate of $D$ symbols/second $\Rightarrow T = 1/D$
- The symbol stream is split into 2 sequences that consist of odd and even symbols, say, $a_n^I$ and $a_n^Q$, respectively
- Let $a_n^I \in \mathcal{M}$ modulate the in-phase carrier $A_c \cos(2\pi f_c t)$ every $T$ seconds to produce the signal
  \[
  A_c \sum_{n=-\infty}^{\infty} a_n^I w(t - nT) \cos(2\pi f_c t) = A_c I(t) \cos(2\pi f_c t)
  \]
- This signal is identical to the BPSK signal if $a_n^I$ is a polar binary symbol sequence
- Similarly, let $a_n^Q \in \mathcal{M}$ modulate the quadrature carrier $A_c \sin(2\pi f_c t)$ every $T$ seconds to produce the signal
  \[
  A_c \sum_{n=-\infty}^{\infty} a_n^Q w(t - nT) \sin(2\pi f_c t) = A_c Q(t) \sin(2\pi f_c t)
  \]
Quadrature Modulation Schemes (contd)

- \(v(t)\) and \(w(t)\) are unit energy pulses of width \(T\) seconds. For example \(v(t) = w(t) = (1 / \sqrt{T}) \Pi[(t - nT) / T]\)
- Both modulated waveforms will have their power spectrum located within the same frequency band
- The composite modulated signal \(x(t)\) is

\[
x(t) = A_c \left[ I(t) \cos(2\pi f_c t) - Q(t) \sin(2\pi f_c t) \right]
\]

\[
= A_c \sum_{n=-\infty}^{\infty} \left[ a_n^I v(t-nT) \cos(2\pi f_c t) - a_n^Q w(t-nT) \sin(2\pi f_c t) \right]
\]

\[\text{PAM} \quad I(t) = \sum_{n=-\infty}^{\infty} a_n^I v(t-nT) \]

\[\text{PAM} \quad Q(t) = \sum_{n=-\infty}^{\infty} a_n^Q w(t-nT) \]

PAM = Pulse amplitude modulation
Quadrature Modulation Schemes (contd)

- The in-phase and quadrature pulse trains $I(t)$ and $Q(t)$ can be recovered by, respectively, multiplying $x(t)$ with $2\cos(2\pi f_c t)$ and $2\sin(2\pi f_c t)$ and then LP filtering resultant waveforms.

- The $M$-ary symbols $a_n^I$ and $a_n^Q$ are then detected from $I(t)$ and $Q(t)$, respectively, as discussed in Chapter 10.

Received signal during the $n$th symbol period

\[ a_n^I \cos(2\pi f_c t) - a_n^Q \sin(2\pi f_c t) \]

\[ 2a_n^I \cos(2\pi f_c t) \]

\[ 2a_n^I \cos^2(2\pi f_c t) - 2a_n^Q \cos(2\pi f_c t) \sin(2\pi f_c t) \]

\[ = a_n^I [1 + \cos(4\pi f_c t) - a_n^Q \sin(4\pi f_c t)] \]

Filtered out

\[ -2a_n^I \cos(2\pi f_c t) \sin(2\pi f_c t) + 2a_n^Q \sin^2(2\pi f_c t) \]

\[ = -a_n^I \sin(4\pi f_c t) + a_n^Q [\cos(4\pi f_c t) + 1] \]

Filtered out
Quaternary Phase Shift Keying (QPSK)

- QPSK is the most common form of phase-shift keying. By using phase shifts of 45, 135, 225, or 315 degrees, each modulated carrier pulse transmits 2 bits of information.
- QPSK is a quadrature modulation scheme: each orthogonal carrier is modulated by a statistically independent polar NRZ symbol sequence.
- The block diagram of a QPSK modulator is shown in Figure.
- Binary data arriving at rate $R_b$ is split by a serial to parallel converter into two data streams, one containing even bits ($b_{2n}$) and other odd bits ($b_{2n+1}$).
- The symbol mapping tables in the upper and lower branches of the modulator encode even and odd bits into polar transmission symbols $a_{2n}$ and $a_{2n+1}$, respectively.
QPSK Modulator

- The output of the pulse shaping filter in the upper branch is a binary polar NRZ pulse train $I(t)$ that modulates the in-phase carrier $A_c \cos(2\pi f_c t)$
- Similarly, a binary polar NRZ pulse train $Q(t)$ generated by the pulse shaping filter in the lower branch modulates the quadrature carrier $A_c \sin(2\pi f_c t)$
- The QPSK signal $x(t)$ is now obtained by adding the in-phase and quadrature components

$$x(t) = A_c \left[ I(t) \cos(2\pi f_c t) - Q(t) \sin(2\pi f_c t) \right]$$

where

$$I(t) = \sum_{n=-\infty}^{\infty} a_{2n} \nu(t - nT)$$

$$Q(t) = \sum_{n=-\infty}^{\infty} a_{2n+1} \nu(t - nT)$$
QPSK Modulator Block Diagram

SPC = Serial-to-parallel converter
SM = Symbol mapping
PSF = Pulse shaping filter
The carrier-modulated pulse during the first symbol interval is
\[ s(t) = A_c v(t) \left[ a_0 \cos(2\pi f_c t) - a_1 \sin(2\pi f_c t) \right], \quad 0 \leq t \leq T \]

\[ = A_c v(t) \cos(2\pi f_c t + \psi_0), \quad \psi_0 = \tan^{-1} \frac{a_1}{a_0} \]

where phase of the transmitted carrier burst \( \psi_0 \) is a discrete random variable assuming one of the four possible values \( \{ \pi/4, 3\pi/4, 5\pi/4, 7\pi/4 \} \) depending on the binary pair \( (a_0, a_1) \).
QPSK Demodulator Block Diagram

- The coherent demodulation of the QPSK signal is shown in Figure.

---

\[
r(t) = x(t) + n(t)
\]

\[
BPF
\]

\[
\cos(2\pi f_c t)
\]

\[
90°
\]

\[
\sin(2\pi f_c t)
\]

\[
I(t) + \text{noise}
\]

\[
\int_0^T\]

\[
\text{Threshold comparator}
\]

\[
\hat{a}_{2n}
\]

\[
\hat{a}_{2n+1}
\]

\[
\text{Binary data } b_n
\]

SM = Symbol mapping

PSC = Parallel-to-serial converter
Error Performance

- By choosing the basis functions
  \[ \phi_1(t) = \sqrt{2}v(t)\cos(2\pi f_c t) \]
  \[ \phi_2(t) = \sqrt{2}v(t)\sin(2\pi f_c t) \]

it is possible express all four possible carrier bursts in table as vectors in the plane spanned by \( \phi_1 \) and \( \phi_2 \)

\[ s = (a_{2n}\sqrt{E_s}, a_{2n+1}\sqrt{E_s}) = (\pm \sqrt{\frac{E_s}{2}}, \pm \sqrt{\frac{E_s}{2}}) = (\pm \sqrt{E_b}, \pm \sqrt{E_b}) \]

- The nearest neighbor estimate for the BER of for QPSK is (\( K = 4, M =4 \))

\[ BER_{QPSK} = \frac{2K}{M \log_2 M} Q \left( \frac{d_{\min}}{\sqrt{2N_o}} \right) = Q \left( \sqrt{\frac{2E_b}{N_o}} \right) \]

\[ E_b = \frac{E_s}{2} \]
\[ d_{\min} = 2\sqrt{E_b} \]

It is exact value
Offset QPSK (OQPSK)

- OQPSK is a minor but important variation on QPSK.
- In QPSK, there is no constraint on allowed phase transitions (0, 90 or 180 degrees as shown by dotted lines) as shown in Figure.
  - $I(t)$ and $Q(t)$ in QPSK can switch signs simultaneously (e.g. if 11 is followed by 00) $\Rightarrow$ the phase $\psi(t)$ changes by $180^\circ$.
- Constant envelope nature of the QPSK signal destroyed with the filtered pulses - the waveform can’t change instantaneously from one peak to another when $180^\circ$ phase transitions occur.
- However, Class-C amplifiers are highly nonlinear and restore the filtered sidelobes causing adjacent channel interference, when amplifying a waveform with envelope variation.
- In OQPSK, either $a_{2n}$ or $a_{2n+1}$ can change but not both because of a single bit delay in the quadrature path $\Rightarrow \pm 90^\circ$ phase transitions only to adjacent neighbors. Less envelope variation.
OQPSK Modulator

- Even bits 00110
- Odd bits 10111
- Delay $T_b$
- $a_{2n}$
- $a_{2n+1}$
- $A_c \cos(2\pi f_c t)$
- $A_c \sin(2\pi f_c t)$
- $I(t) = \sum_{n=-\infty}^{\infty} a_{2n}v(t-nT)$
- $Q(t) = \sum_{n=-\infty}^{\infty} a_{2n+1}v(t-nT)$

SPC = Serial-to-parallel converter
SM = Symbol mapping
PSF = Pulse shaping filter

(a) QPSK
(b) OQPSK
The OQPSK demodulator is identical to that of QPSK demodulator except for a single bit delay in the inphase path.

Since OQPSK constellation is identical to that of QPSK, its BER performance is identical to that of QPSK.
**M-ary Phase Shift Keying**

- In *M*-ary PSK, *M* different phase shifts of the carrier are used to convey the information. The \( M = 2^k \) signal waveforms, each representing *k* information bits, are represented as

\[
s_i(t) = A_c \nu(t) \cos[2\pi f_c t + \psi_i + \varphi], \quad 0 \leq t \leq T \quad i = 1, \ldots, M
\]

where

- \( \varphi = 0 \) or \( \frac{\pi}{M} \) = Fixed phase offset
- \( \psi_i = \frac{2\pi(i-1)}{M} \) = *M* possible phases of the carrier
- \( \nu(t) \) = unit energy pulse

\[x(t) = \sqrt{\frac{2E_s}{T}} \sum_{n=-\infty}^{\infty} \nu(t - nT) \cos[2\pi f_c t + \psi_n + \varphi]\]

*All M-ary PSK waveforms have equal energy* \( E_s \)

*Phase of the carrier during the* \( n \)th symbol interval
**M-ary PSK (contd)**

- By choosing the same basis functions as for QPSK, it is possible to express all waveforms in the $M$-PSK signal set as vectors in the plane spanned by $\phi_1$ and $\phi_2$ as

\[
s = (a_n^I \sqrt{E_s}, a_n^O \sqrt{E_s})
\]

where

\[
a_n^I = \cos(\psi_n + \varphi)
\]

\[
a_n^O = \sin(\psi_n + \varphi)
\]

- The signal vectors lie around a circle of radius $\sqrt{E_s}$. The constellation for 8-PSK ($M = 8$) is shown in Figure
**M-ary PSK (contd)**

- As illustrated in Figure, the minimum distance between two adjacent signal points is

  \[ d_{\text{min}} = 2D = 2\sqrt{E_s} \sin \left( \frac{\pi}{M} \right) \]

- The nearest-neighbor estimate of \( P_e \) is

  \[
  P_e \approx 2Q \left[ \sqrt{\frac{2E_s}{N_o} \sin^2 \left( \frac{\pi}{M} \right)} \right]
  \]

  \[
  = 2Q \left[ \sqrt{\frac{2E_b \log_2 M}{N_o} \sin^2 \left( \frac{\pi}{M} \right)} \right]
  \]

  \[
  \text{BER}_{\text{MPSK}} \approx \frac{1}{\log_2 M} 2Q \left[ \sqrt{\frac{2E_b \log_2 M}{N_o} \sin^2 \left( \frac{\pi}{M} \right)} \right]
  \]
M-PSK BER Performance
# Digital Carrier Modulation Schemes

<table>
<thead>
<tr>
<th>Binary BP Signaling</th>
<th>Null-to-Null RF Bandwidth (Hz)</th>
<th>Abs-Abs Bandwidth (Hz)</th>
<th>BER with Coherent Detection</th>
<th>BER with Noncoherent Detection</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASK</td>
<td>$2R_b$</td>
<td>$R_b(1-\alpha)$</td>
<td>$Q(\sqrt{E_b/N_o})$</td>
<td>$0.5e^{-R_b/2N_o}$</td>
</tr>
<tr>
<td>BPSK</td>
<td>$2R_b$</td>
<td>$R_b(1-\alpha)$</td>
<td>$Q(\sqrt{2E_b/N_o})$</td>
<td>Requires coherent detection</td>
</tr>
<tr>
<td>Sunde’s FSK</td>
<td>$3R_b$</td>
<td>$R_b(1-\alpha)$</td>
<td>$Q(\sqrt{E_b/N_o})$</td>
<td>$0.5e^{-R_b/2N_o}$</td>
</tr>
<tr>
<td>DBPSK</td>
<td>$2R_b$</td>
<td>$R_b(1-\alpha)$</td>
<td></td>
<td>$0.5e^{-R_b/2N_o}$</td>
</tr>
<tr>
<td>M-ary BP Signaling</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>QPSK/OQPSK</td>
<td>$R_b$</td>
<td>$R_b(1-\alpha)/2$</td>
<td>$Q(\sqrt{2E_b/N_o})$</td>
<td>Requires coherent detection</td>
</tr>
<tr>
<td>MSK</td>
<td>$1.5R_b$</td>
<td>$3R_b(1-\alpha)/4$</td>
<td>$Q(\sqrt{2E_b/N_o})$</td>
<td>Requires coherent detection</td>
</tr>
<tr>
<td>M-PSK (M=4)</td>
<td>$2R_b/\log_2 M$</td>
<td>$R_b(1-\alpha)/\log_2 M$</td>
<td>$\frac{2}{\log_2 M}Q(\sqrt{2\log_2 M}\sin^2(\pi/M)E_b/N_o}$</td>
<td>Requires coherent detection</td>
</tr>
<tr>
<td>M-DPSK (M=4)</td>
<td>$2R_b/\log_2 M$</td>
<td>$R_b(1-\alpha)/\log_2 M$</td>
<td></td>
<td>$\frac{2}{\log_2 M}Q(\sqrt{4\log_2 M}\sin^2(\pi/2M)E_b/N_o)$</td>
</tr>
<tr>
<td>M-QAM (Square constellation)</td>
<td>$2R_b/\log_2 M$</td>
<td>$R_b(1-\alpha)/\log_2 M$</td>
<td>$\frac{4}{\log_2 M}(1-\frac{1}{\sqrt{M}})Q(\sqrt{3\log_2 M}E_b/N_o/(M-1))$</td>
<td>Requires coherent detection</td>
</tr>
<tr>
<td>M-FSK Coherent Noncoherent</td>
<td>$(M+3)R_b/2\log_2 M$</td>
<td>$2M R_b/\log_2 M$</td>
<td>$\frac{M-1}{\log_2 M}Q(\sqrt{(\log_2 M)E_b/N_o})$</td>
<td>$\frac{M-1}{2\log_2 M}0.5e^{-(\log_2 M)R_b/2N_o}$</td>
</tr>
</tbody>
</table>