Chapter 5

AM Modulation
Outline

• AM Modulation
AM Modulation

- In order to transfer signals we need to transfer the frequency to higher level
- One approach is using modulation
- Modulation:
  - Changing the amplitude of the carrier
- AM modulation is one type of modulation
  - Easy, cheap, low-quality
  - Used for AM receiver and CBs (citizen bands)
  - Generally high carrier frequency is used to modulate the voice signal (300 – 3000 Hz)
AM Modulation

• In AM modulation the carrier signal changes (almost) linearly according to the modulating signal - m(t)
• AM modulating has different schemes
  – Double-sideband suppressed carrier (DSB-SC)
  – Double-sideband Full Carrier (DSB-FC)
    • Also called the Ordinary AM Modulation (AM)
  – Single-sideband (SSB)
  – Vestigial Sideband (VSB) – Not covered here!
Assuming the Modulating Signal is Sinusoid
AM Modulation

Vm is the modulating signal.
In this case:

- $V_c(t) = E_c \sin \omega_c t$
- $V_m(t) = E_m \sin \omega_m t$
- $V_{\text{AM}}(t) = E_c \sin \omega_c t + E_m \sin \omega_m t \cdot \sin \omega_c t$

Assume $E_m = mE_c$; where $0 < m < 1 \Rightarrow m$ is called the modulation index, or percentage modulation!
• Rearranging the relationship:

\[ v_{am}(t) = E_c \sin(2\pi f_c t) + [mE_c \sin(2\pi f_m t)][\sin(2\pi f_c t)] \]

\[ v_{am}(t) = E_c \sin(2\pi f_c t) - \frac{mE_c}{2}\cos[2\pi(f_c + f_m)t] + \frac{mE_c}{2}\cos[2\pi(f_c - f_m)t] \]

• This Carrier + LSB + USB

• Note that
  – \( V_{am}(\max = E_c + mE_c = 2E_c ; \text{ for } m = 1 \)
  – \( V_{am}(\min = 0 \text{ ; for } m=1 \)
Phase Difference

\[ v_{am}(t) = E_c \sin(2\pi f_c t) - \frac{mE_c}{2} \cos[2\pi(f_c + f_m)t] + \frac{mE_c}{2} \cos[2\pi(f_c - f_m)t] \]
AM Modulation

$s_{AM}(t) = m(t)\cos\omega_c t + A\cos\omega_t$

Envelope of $s_{AM}(t)$

$A > |m(t)|_{max}$

$A < |m(t)|_{max}$
AM Power Distribution

- \( P = E^2/2R = Vp^2/2R \); \( R \) = load resistance
- Remember: \( P_{\text{avg}} = \frac{V_{\text{rms}}^2}{R} \); where \( V_{\text{rms}} \) for sinusoidal is \( V_p/\sqrt{2} \)

\[
v_{\text{am}}(t) = E_c \sin(2\pi f_c t) - \frac{mE_c}{2} \cos[2\pi(f_c + f_m)t] + \frac{mE_c}{2} \cos[2\pi(f_c - f_m)t]
\]

- \( P_{\text{carrier\_average}} = \frac{E_c^2}{2R} \)
- \( P_{\text{usb\_average}} = \frac{(mE_c/2)^2}{2R} = \frac{(m^2/4)P_c}{4} \)
- \( P_{\text{total}} = P_{\text{carrier\_average}} + P_{\text{usb\_average}} + P_{\text{lsb\_average}} \)

What happens as \( m \) increases?
Current Analysis

• Measuring output voltage may not be very practical
• \( P = \frac{Vp^2}{2R} \) is difficult to measure in an antenna!
• However, measuring the current passing through an antenna may be more possible: Total Power is \( P_T = I_T^2R \)

\[
\frac{P_i}{P_c} = \frac{I_i^2R}{I_c^2R} = \frac{I_i^2}{I_c^2} = 1 + \frac{m^2}{2}
\]

\[
\frac{I_i}{I_c} = \sqrt{1 + \frac{m^2}{2}}
\]

\[
I_t = I_c \sqrt{1 + \frac{m^2}{2}}
\]

Note that we can obtain \( m \) if we measure currents!
Multiple Input Frequencies

• What if the modulating signal has multiple frequencies?

\[ v_{am}(t) = \sin(2\pi f_c t) + \frac{1}{2} \cos[2\pi (f_c - f_{m1}) t] - \frac{1}{2} \cos[2\pi (f_c + f_{m1}) t] \]

\[ + \frac{1}{2} \cos[2\pi (f_c - f_{m2}) t] - \frac{1}{2} \cos[2\pi (f_c + f_{m2}) t] \]

• In this case:

\[ m_t = \sqrt{m_1^2 + m_2^2 + m_3^2 + m_4^2} \]

• All other power measurements will be the same!
Examples (5A, 5C)
General Case: $m(t)$ can be any bandpass
Review: Bandpass Signal

- Remember for bandpass waveform we have

\[ s(t) = \text{Re}\{g(t)e^{j\omega t}\} \]

- The voltage (or current) spectrum of the bandpass signal is

\[ S(f) = \frac{1}{2}[G(f - f_c) + G^*(-f - f_c)] \]

- The PSD will be

\[ P_s(f) = \frac{1}{4} [P_g(f - f_c) + P_g(-f - f_c)] \]

- In case of Ordinary AM (DSB – FC) modulation:

\[ g(t) = A_c[1 + m(t)] \]

- In this case Ac is the power level of the carrier signal with no modulation;

- Therefore:

\[ s(t) = A_c[1 + m(t)] \cos \omega_c t \]

Make sure you know where these come from!
AM: Modulation Index

• Modulation Percentage (m)

\[
\text{\% modulation} = \frac{A_{\text{max}} - A_{\text{min}}}{2A_c} \times 100 = \frac{\max [m(t)] - \min [m(t)]}{2} \times 100
\]

• Note that \(m(t)\) has peak amplitude of \(A_m = mE_m = mA_c\)

• We note that for ordinary AM modulation,
  - if the modulation percentage > %100,
  - implying \(m(t) < -1\)
  - Then:

\[
s(t) = \begin{cases} 
A_c[1 + m(t)] \cos \omega t, & \text{if } m(t) \geq -1 \\
0, & \text{if } m(t) < -1
\end{cases}
\]
This is how we generate the ordinary AM using MATLAB

```matlab
fc = 10; % carrier frequency
fa = 1; % modulating frequency
N = 200; % number of samples
To = 4; % observation time: T0 x periods
MI = 1; % Modulation Index (0.0-2.0 or 0 to 200 percent)
Ec = 1; % Ec is the level of the AM envelope in the absence of modulation, when m(t) = 0;

Ta = 1/fa;
dt = To*Ta/N;
wc = 2*pi*fc;
wa = 2*pi*fa;

t = 0:dt:To*Ta; % simulation time

m = MI*cos(wa*t); % modulating signal: m(t)
m = m(:);

y = zeros(length(t),1); % In this part we force [1+m] = 0 if
for (i = 1:1:length(t)) %
    if (m(i) > -1) % in other words, we ensure [1+m(t)]=0 if
        y(i) = 1; % m(t) < -1
    end;
end;
end;
```
AM: Normalized Average Power

- Normalized Average Power (R=1)
- Note that
  \[ \langle s^2(t) \rangle = \frac{1}{2} A_c^2 + \frac{1}{2} A_c^2 \langle m^2(t) \rangle \]
  = \frac{1}{2} A_c^2 (1 + 2m(t) + m^2(t))
  = \frac{1}{2} A_c^2 + A_c^2 \langle m(t) \rangle + \frac{1}{2} A_c^2 \langle m^2(t) \rangle

- Pc is the normalized carrier power \((1/2)A_c^2\) (when R= 1, Ac = Ec, and m is the modulation index)
- The rest is the power of each side band
- Thus:

\[ \frac{1}{2} \cdot A_c \]

\[ P_{usf} = \frac{(1/2 \cdot A_c \cdot \langle m(t)^2 \rangle)}{2} \]
AM: Modulation Efficiency

- Defined as the percentage of the total power of the modulated signal that conveys information

\[ s(t) = A_c [1 + m(t)] \cos \omega_c t \]

- Defined as:

\[ E = \frac{\langle m^2(t) \rangle}{1 + \langle m^2(t) \rangle} \times 100\% \]

- Normalized Peak Envelop Power is defined as

\[ P_{PEP} = \frac{(A_c^2)}{2} \times (1 + A_{max})^2 = \]

(when load resistance R=1)

- We use \( P_{PEP} \) to express transmitter output power.
- In general, Normalized Peak Envelop Power, \( P_{PEP} \), can be expressed as follow:

\[ \frac{1}{2} \max \{ |g(t)|^2 \} \]
AM: Voltage and Current Spectrum

- We know for AM:  
  \[ s(t) = A_c[1 + m(t)] \cos \omega_c t \]
- The voltage or Current Spectrum will be  
  \[ S(f) = \frac{A_c}{2} [\delta(f - f_c) + M(f - f_c) + \delta(f + f_c) + M(f + f_c)] \]

Note that BW is 2B – doubled compared to M(f) →
1- Large bandwidth requirement
2- Duplicated Information in Upper and Lower Sides
3- We are wasting power to send the discrete carrier power
Building an Ordinary AM Modulator

• Transferring AC power to RF power!

• Two general types
  – Low power modulators
  – High power modulators

• Low Power Modulators
  – Using multipliers and amplifiers
  – Issue: Linear amplifiers must be used; however not so efficient when it comes to high power transfer

• High Power Modulators
  – Using PWM
Building an Ordinary AM Modulator

Using Pulse Width Modulation and Power Amplifiers (Class C)
• Assume $P_{c\text{-avg}} = 5000$ W for a radio station (un-modulated carrier signal); If $m=1$ (100 percent modulation) with modulated frequency of 1KHz sinusoid find the following:
  – Peak Voltage across the load ($A_c$)
  – Total normalized power ($<s(t)^2>$)
  – Total Average (actual) Power
  – Normalized PEP
  – Average PEP
  – Modulation Efficiency – Is it good?
Double Sideband Suppressed Carrier

• DSB-SC is useful to ensure the discrete carrier signal is suppressed:

\[ s(t) = A_c m(t) \cos \omega_c t \]

• The voltage or current spectrum of DSB-SC will be

\[ S(f) = \frac{A_c}{2} \left[ M(f - f_c) + M(f + f_c) \right] \]

• Therefore no waste of power for discrete carrier component!

• What is the modulation efficiency? → 100 Percent!
  – Effic = \( \frac{\langle m(t)^2 \rangle}{\langle m(t)^2 \rangle} \)

• Generating DSB-SC

\[ s_{\text{DSB-SC}}(t) = m(t) \cos \omega_c t \]

\[ e(t) \]

Low-pass Filter

\[ s_d(t) \]
Multiplying the signal $m(t) \cos \omega_c t$ by a **local carrier wave** $\cos \omega_c t$

$e(t) = m(t) \cos^2 \omega_c t = (1/2)[m(t) + m(t)\cos 2\omega_c t]$

$E(\omega) = (1/2)M(\omega) + (1/4)[M(\omega + 2\omega_c) + M(\omega - 2\omega_c)]$

Passing through a **low pass filter**:

$S_o(\omega) = (1/2)M(\omega)$

The output signal:

$s_o(t) = (1/2)m(t)$
DSB-SC

Modulator

\[ m(t) \rightarrow s_{\text{DSB-SC}}(t) = m(t) \cos \omega_c t \]

\[ \cos \omega_c t \]

\[ m(t) \]

\[ t \]

\[ t \]

\[ \omega \]

\[ \omega_m \]

\[ -\omega_m \]

A

\[ M(\omega) \]

\[ E(\omega) \]

\[ A/2 \]

\[ A/4 \]

\[ 2\omega_c \]

\[ -2\omega_c \]

\[ 2\omega_m \]

\[ \omega_m \]

\[ -\omega_m \]

\[ \omega \]

\[ S_{\text{DSB-SC}}(\omega) \]

Lower side band

Upper side band

\[ 2\omega_c \]

\[ 0 \]

\[ s_c(t) \]

Low-pass Filter

\[ s_c(t) \]
DSB-SC – Coherent Demodulation Issues

So what if the Local oscillator frequency is a bit off with the center frequency ($\Delta\omega$)?

\[ s_{\text{DSB-SC}}(t) = m(t) \cos \omega_c t \]

Multiplying the signal $m(t)\cos \omega_c t$ by a local carrier wave $\cos[(\omega_c+\Delta\omega)t]$

\[ e(t) = m(t)\cos \omega_c t \cdot \cos[(\omega_c+\Delta\omega)t] \]
\[ = (1/2)[m(t)] \cdot \{ \cos[\omega_c t -(\omega_c+\Delta\omega)t] + \cos[\omega_c t + (\omega_c+\Delta\omega)t] \} \]
\[ = (1/2)[m(t)] \cdot \{ \cos(\Delta\omega t) + \cos(2\omega_c+\Delta\omega)t \} \]
\[ = m(t)/2 \cdot \cos(\Delta\omega t) \]

The coherent demodulator must be synchronized with the modulator both in frequency and phase!

Disadvantages:

1. It transmits both sidebands which contain identical information and thus waste the channel bandwidth resources;
2. It requires a fairly complicated (expensive) circuitry at a remotely located receiver in order to avoid phase errors.
Demodulation DSB-SC

- One common approach is using Squaring Loop:

\[ s(t) = A_c m(t) \cos(\omega_c t) \]
\[ s_2(t) = \frac{1}{2} A_c^2 m^2(t) [1 + \cos(2\omega_c t)] \]
\[ \frac{1}{2} A_c m^2(t) \cos(2\omega_c t) \]
\[ A_0 \cos(\omega_c t) \]
\[ \frac{1}{2} A_c A_0 m(t) \]

Note that in this case the initial phase must be known!
Single Sideband AM (SSB)

- Is there anyway to reduce the bandwidth in ordinary AM?
- The complex envelop of SSB AM is defined by
  \[ g(t) = A_c[m(t) \pm j\hat{m}(t)] \]
- Thus, we will have
  \[ s(t) = A_c[m(t) \cos \omega_c t \mp \hat{m}(t) \sin \omega_c t] \]
- In this case the (+) $\rightarrow$ USSB and (-) $\rightarrow$ LSSB
- We define ($\sim m(t)$ is the Hilbert Transfer of $m(t)$)
  \[ \hat{m}(t) \triangleq m(t) \ast h(t) \]

  - Where:
  - With
  - Thus:

  \[
  H(f) = \begin{cases}
  -j, & f > 0 \\
  j, & f < 0
  \end{cases}
  \]

  \[
  G(f) = A_c\{M(f) \pm jF[\hat{m}(t)]\} \quad \Rightarrow \quad G(f) = A_cM(f)[1 \pm jH(f)]
  \]
Frequency Spectrum of SSB-AM - USSB

For Upper SSB use (+)

\[ G(f) = A_c M(f) [1 \pm jH(f)] \]

\[ H(f) = \begin{cases} -j, & f > 0 \\ j, & f < 0 \end{cases} \quad \rightarrow \quad G(f) = \begin{cases} 2A_c M(f), & f > 0 \\ 0, & f < 0 \end{cases} \]

Therefore:

\[ s(t) = A_c [m(t) \cos \omega_c t \mp \hat{m}(t) \sin \omega_c t] \]

\[ S(f) = A_c \begin{cases} M(f - \bar{f}_c), & f > \bar{f}_c \\ 0, & f < \bar{f}_c \end{cases} + A_c \begin{cases} 0, & f > -\bar{f}_c \\ M(f + \bar{f}_c), & f < -\bar{f}_c \end{cases} \]

Normalized Average Power:

\[ \langle s^2(t) \rangle = \frac{1}{2} \langle |g(t)|^2 \rangle = \frac{1}{2} A_c^2 \langle m^2(t) + \hat{m}(t)^2 \rangle \quad \langle \hat{m}(t)^2 \rangle = \langle m^2(t) \rangle \]

\[ \langle s^2(t) \rangle = A_c^2 \langle m^2(t) \rangle \]
(a) Baseband Magnitude Spectrum

(b) Magnitude of Corresponding Spectrum of the Complex Envelope for USBB

(c) Magnitude of Corresponding Spectrum of the USBB Signal
Phasic Method

\[ s(t) = A_c [m(t) \cos \omega_c t \mp \hat{m}(t) \sin \omega_c t] \]

This is also called Quadrature AM (QAM) modulator with I and Q channels.
AM Modulators: Frequency Multiplier

Nonlinear amplifier and a filter to extract the nth harmonic!
Building AM Modulators

- AM Modulating Circuits are categorized as
  - Low-level Transmitters
  - Medium-level Transmitters
  - High-level Transmitters
Other Key Components

• Mixers
• Phase shifter
  – RC
  – Inverters
• Amplifiers
  – Linear
  – Nonlinear
Low-Level AM Modulators

- Requires less modulating signal power to achieve high $m$
- Mainly for low-power applications
- Uses an **Emitter Modulator** (low power)
  - Incapable of providing high-power
- The amplifier has two inputs: $V_c(t)$ and $V_m(t)$
- The amplifier operates in both linear and nonlinear modes
Low-Level AM Modulators – Circuit Operation

- If \( V_m(t) = 0 \rightarrow \) amplifier will be in **linear** mode
  - \( \rightarrow A_{out} = V_c \cos(w_c t) \); \( V_c \) is voltage gain (unit less)
- If \( V_m(t) > 0 \rightarrow \) amplifier will be in **nonlinear** mode
  - \( \rightarrow A_{out} = [V_c + V_m \cos(w_c t)] \cos(w_c t) \)
- \( V_m(t) \) is isolated using T1
  - The value of \( V_m(t) \) results in Q1 to go into cutoff or saturation modes
- C2 is used for coupling
  - Removes modulating frequency from AM waveform
High-Level AM Modulators – Circuit Operation

• Used for high-power transmission
• Uses an **Collector Modulator** (high power)
  – Nonlinear modulator
• The amplifier has two inputs: $V_c(t)$ and $V_m(t)$
• **RFC** is radio frequency choke
  – blocks RF
High-Level AM Modulators – Circuit Operation

- General operation:
  - If Base Voltage > 0.7 $\rightarrow$ Q1 is ON $\rightarrow$ $I_c \neq 0$ $\rightarrow$ Saturation
  - If Base Voltage < 0.7 $\rightarrow$ Q1 is OFF $\rightarrow$ $I_c = 0$ $\rightarrow$ Cutoff
  - The Transistor changes between Saturation and Cutoff
- When in nonlinear $\rightarrow$ high harmonics are generated $\rightarrow$ $V_{out}$ must be bandlimited
High-Level AM Modulators – Circuit Operation

- $C_L$ and $L_L$ tank can be added to act as Bandlimited
  - Only $f_c + f_m$ and $f_c - f_m$ can be transmitted
AM Modulators – Using Integrated Devices

- XR-2206 is an integrated circuit function generator
- In this case $f_c = \frac{1}{R_1 C_1}$ Hz
- For example in this case: if $f_m = 4$kHz; $f_c = 100$kHz
AM Demodulators: Envelope Detector

- Considered as **non-coherent** demodulators
- The diode acts as a **nonlinear** mixer
- Other **names**
  - Diode Detector
  - Peak Detector (Positive)
  - Envelope Detector
- Basic operation: Assume $f_c = 300$ KHz and $f_m = 2$KHz
  - Then there will be frequencies $298$, $300$, $302$ KHz
  - The detector will detect many different frequencies
  - **AM frequencies + AM harmonics + SUM of AM frequencies + DIFF of AM frequencies**
  - The RC LPF is set to pass only DIFF frequencies
Envelope Detector – Basic Operation

- The diode has $V_{\text{barrier}} = V_b = 0.3V$
- When $V_{\text{in}} < V_b \rightarrow$ Reverse Biased
  $\rightarrow$ DIODE is OFF
  $\rightarrow i_d = 0 \rightarrow V_{\text{cap}} = 0$
- When $V_{\text{in}} > V_b \rightarrow$ Forward Biased
  $\rightarrow$ DIODE is ON
  $\rightarrow i_d > 0 \rightarrow V_{\text{cap}} = V_{\text{in}} - 0.3$
Envelope Detector – Distortion

• What should be the value of RC?
  – If too low then discharges too fast
  – If too high the envelope will be distorted
  – The highest modulating signal:

\[ f_{m(\text{max})} = \frac{\sqrt{1 / m^2} - 1}{2\pi RC} \]

  – Note that in most cases m=0.70 or 70 percent of modulation

\[ f_{m(\text{max})} = \frac{1}{2\pi RC} \]

\[ B \ll \frac{1}{2\pi RC} \ll f_c \]
Standard (Ordinary) AM

**AM signal generation**

\[ s_{AM}(t) = [A + m(t)] \cos \omega_c t \]

**Waveform** :

\[ s_{AM}(t) = A \cos \omega_c t + m(t) \cos \omega_c t = [A + m(t)] \cos \omega_c t \]

**Spectrum** :

\[ S_{AM}(\omega) = \frac{1}{2} [M(\omega + \omega_c) + M(\omega - \omega_c)] + \pi A [\delta(\omega + \omega_m) + \delta(\omega - \omega_m)] \]
Standard (Ordinary) AM

• The disadvantage of high cost receiver circuit of the DSB-SC system can be solved by use of AM, but at the price of a less efficient transmitter

• An AM system transmits a large power carrier wave, \( A \cos \omega_c t \), along with the modulated signal, \( m(t) \cos \omega_c t \), so that there is no need to generate a carrier at the receiver.
  – Advantage: simple and low cost receiver

• In a broadcast system, the transmitter is associated with a large number of low cost receivers. The AM system is therefore preferred for this type of application.
References