Vectors and Force

Purpose
The objective of this lab is to understand vectors and practice adding and resolving vectors.

Concepts
Physical quantities are classified as either scalar or vector quantities. A scalar quantity is one with magnitude only. On the other hand, a vector quantity has both magnitude and direction. Common examples of scalar quantities are (distance, speed, temperature and energy). (Displacement, velocity, acceleration and momentum) are examples of vector quantities. This week’s topic is one of the most important concepts in physics, Force. The force is a vector quantity. For better understanding of force, we will practice vector analysis. A bold character or an arrow on top of a character indicates a vector quantity. (Example: $A$, $B$, $C$ or $\vec{A}, \vec{B}, \vec{C}$) Commonly vectors are represented by arrows graphically or by components. The length of a vector arrow is proportional to the magnitude of the vector, and the arrow points in the direction of the vector. Scalar quantities are added (or subtracted) simply by adding (or subtracting) numbers. On the other hand, the addition and resolution of vectors are not as easy since they are associated with directions of vectors. Parallelogram method, Head to tail method and component method are three commonly used methods for vector addition. When two vectors $A + B$ are added to form the resultant vector, $C$, the direction of $C$ may be specified as being at an angle $\theta$ relative to $A$.

\[
C_x = A_x + B_x \\
C_y = A_y + B_y \\
C = \sqrt{C_x^2 + C_y^2} \\
\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right)
\]

Equipment

<table>
<thead>
<tr>
<th>Vector and Force table</th>
<th>Hooke’s Law</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force Table</td>
<td>Spring</td>
</tr>
<tr>
<td>Hangers (x4) and weights</td>
<td>String</td>
</tr>
<tr>
<td>Pulley, String</td>
<td>Weights</td>
</tr>
<tr>
<td>Protractors and Ruler</td>
<td>Ruler</td>
</tr>
</tbody>
</table>

Part 1. [Force Table and Vector Addition]
The force table consists of a circular table (which is calibrated in degrees), a ring in the center of the table, pulleys and weight hangers. The magnitude of a force (vector) is varied by adding or removing weights (mg), and the direction is varied by moving the pulley around the table. The resultant force is found by balancing the forces with another force (weights on a hanger) so that the ring is centered around the central pin. The balancing force is not the resultant force ($C$), but rather the equilibrant ($-C$) that balances the other forces in mechanical equilibrium. The equilibrant is the vector force of equal magnitude, but in the opposite direction to the resultant. To get the direction of the resultant force ($\theta$), $180^\circ$ should be subtracted from the angle of the equilibrant.
Procedure
1. Set up the force table with strings and suspended 3 hangers.
2. Measure the mass of each hanger.
3. Clamp pulleys at 30° and 120°. Hang 150g weights at the end of each string. (mass of hanger + mass of weights should be 150g).
4. Compute the corresponding force on each string in Newtons. (F=total mass in kg x 9.8)
5. Using a third pulley and weights, determine the magnitude and direction of the equilibrant force that maintains the central ring centered in equilibrium around the center of the force table. Record the magnitudes and directions of the equilibrant force (F_{\text{tot}}) and the resultant force (F_{\text{tot}}).

Data and Analysis

<Vector addition 1>
\[ \vec{F}_1 + \vec{F}_2 = \vec{F}_{\text{tot}}, \quad \vec{F}_1 : \text{(150g at 30°)}, \quad \vec{F}_2 : \text{(150g at 120°)} \]

<table>
<thead>
<tr>
<th>Measurement</th>
<th>F1</th>
<th>F2</th>
<th>-F_{\text{tot}}</th>
<th>Resultant F_{\text{tot}}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of hanger [g]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angles</td>
<td>30°</td>
<td>120°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass of weights [g]</td>
<td>150g</td>
<td>150g</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Force [N]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Analytical Method)

6. Draw a vector diagram using arrows. \( \vec{F}_1 + \vec{F}_2 = \vec{F}_{\text{tot}} \)
7. Compute x and y components of \( \vec{F}_1 \) and \( \vec{F}_2 \).
8. Add x component of \( \vec{F}_1 \) and \( \vec{F}_2 \) to obtain the x component of the resultant.
9. Compute the y component of the resultant.
10. Compute the magnitude of the resultant. (\( F_{\text{tot}} = \sqrt{F_{\text{tot,x}}^2 + F_{\text{tot,y}}^2} \))
11. Compute the angle of the resultant. (\( \theta = \tan^{-1}\left(\frac{F_{\text{tot,y}}}{F_{\text{tot,x}}}\right) \))

<table>
<thead>
<tr>
<th></th>
<th>X component</th>
<th>Y component</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F_{\text{tot}}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F_{\text{tot,magnitude}}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F_{\text{tot,angle}} (\Theta)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Are the analytical values agreed with experimental values? __________
Vector addition 2

\[
\vec{F}_1 + \vec{F}_2 = \vec{F}_{\text{tot}}, \quad \vec{F}_1 : (50 \text{g at } 20^\circ), \quad \vec{F}_2 : (150 \text{g at } 90^\circ)
\]

<table>
<thead>
<tr>
<th>Measurement</th>
<th>F1</th>
<th>F2</th>
<th>(-F_{\text{tot}})</th>
<th>Resultant F_{\text{tot}}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of hanger [g]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angles</td>
<td>20°</td>
<td>90°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass of weights [g]</td>
<td>50g</td>
<td>150g</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Find resultant vector \(\vec{F}_{\text{tot}}\) experimentally.
2. Draw a vector diagram to add two vectors, \(\vec{F}_1\) and \(\vec{F}_2\).
3. Analytically find the resultant vector, \(\vec{F}_{\text{tot}}\).

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{F1} & \text{F2} & \text{F}_{\text{tot}} & \text{F}_{\text{tot,magnitude}} & \text{F}_{\text{tot,angle (\Theta)}} \\
\hline
\end{array}
\]

Are the analytical values agreed with experimental values? 

Vector addition 3

\[
\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{F}_{\text{tot}}, \quad \vec{F}_1 : (50 \text{g at } 20^\circ), \quad \vec{F}_2 : (150 \text{g at } 90^\circ)
\]

<table>
<thead>
<tr>
<th>Measurement</th>
<th>F1</th>
<th>F2</th>
<th>F3</th>
<th>(-F_{\text{tot}})</th>
<th>Resultant F_{\text{tot}}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angles</td>
<td>30°</td>
<td>60°</td>
<td>225°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass of weights [g]</td>
<td>50g</td>
<td>100g</td>
<td>250g</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{F1} & \text{F2} & \text{F3} & \text{F}_{\text{tot}} & \text{F}_{\text{tot,magnitude}} & \text{F}_{\text{tot,angle (\Theta)}} \\
\hline
\end{array}
\]

Are the analytical values agreed with experimental values? 


Part 2. [Hooke's Law]

When a spring is stretched or compressed from the equilibrium position, a recoiling force from the spring tries to restore the spring to the initial equilibrium. Experimentally, the spring’s displacement, \( x \) from the equilibrium position is proportional to the restoring force, \( F \).

\[
F = -kx
\]

Negative sign means the restoring force is opposite from the direction of the displacement. The above equation is known as Hooke’s law. The coefficient \( k \) is called the spring constant and it is an intrinsic property of each spring. In this part, we will obtain the spring constant, \( k \).

1. Start with a spring in a vertical position with a small amount of weight suspended.
2. Draw a free body diagram for the weight.
3. Devise a method to verify Hooke's law.
4. Determine the spring constant, \( k \).