Capacitors

Capacitors are devices that can store electric charge similar to a battery. In its simplest form we can think of a capacitor to consist of two metallic plates separated by air or some other insulating material.

The capacitance of a capacitor is shown by \( C \) (in units of Farad) and indicates the ratio of electric charge \( Q \) accumulated on its plates to the voltage \( V \) across it (\( Q = C V \)). The capacitance itself is strictly a function of the geometry of the device and the type of insulating material that fills the gap between its plates. \( C = (\varepsilon A)/d \), where \( \varepsilon \) is the permittivity of the material in the gap, \( A \) is the area of the plate and \( d \) is the separation between the plates. The formula is more complicated for cylindrical and other geometries. However, it is clear that the capacitance is large when the area of the plates are large and they are closely spaced. In order to create a large capacitance, we can increase the surface area of the plates by rolling them into cylindrical layers as shown in the diagram above.

Note that if the space between plates is filled with air, then \( C = (\varepsilon_0 A)/d \), where \( \varepsilon_0 \) is the permittivity of free space. The ratio of \( (\varepsilon/\varepsilon_0) \) is called the dielectric constant of the material. The range of dielectric constants of some materials is given below.

<table>
<thead>
<tr>
<th>Material</th>
<th>Dielectric Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1</td>
</tr>
<tr>
<td>Bakelight</td>
<td>5 - 22</td>
</tr>
<tr>
<td>Formica</td>
<td>3.6 - 6</td>
</tr>
<tr>
<td>Epoxy Resin</td>
<td>3.4 – 3.7</td>
</tr>
<tr>
<td>Glass</td>
<td>3.8 – 14.5</td>
</tr>
<tr>
<td>Mica</td>
<td>4 - 9</td>
</tr>
<tr>
<td>Paper</td>
<td>1.5 - 3</td>
</tr>
<tr>
<td>Parafin</td>
<td>2 - 3</td>
</tr>
</tbody>
</table>
In order to increase the capacitance of a capacitor, in addition to increasing $A$ and decreasing $d$, we will need a material with a large dielectric constant. On the other hand, in order to prevent the charges on one plate to leak to the other plate, it is important that the dielectric material be a fairly good insulator. Therefore we may not arbitrarily choose any material with a large dielectric constant. Moreover, it is preferred for the material to be flexible so it can be rolled into the space between the cylindrical layers. In general we have two types of capacitors: Ceramic and electrolytic. Ceramic capacitors do not have any polarity which means that the either plate can accumulate positive or negative charges. However, the pins of electrolytic capacitors are marked with their designated polarity and one must be careful to place these capacitors in the circuit with the correct polarity.

![Image of capacitors]

The ability of capacitors to store charges is similar to batteries. However, the main difference between a battery and a capacitor is that capacitors can be charged and discharged very quickly, whereas batteries are meant to discharge very slowly. Note that Farad is a very large unit for capacitance. The capacitance of capacitors in circuits could vary from pico Farad (pF) to mili Farad (mF).

Capacitors are generally used in series and/or parallel with resistors. When a capacitor (in series with a resistor) is connected to a battery, the capacitor’s plates begin to accumulate charge on them (negative on one side and positive on the other). The rate of charging and discharging of capacitors is an exponential function of the resistance of the resistor and the capacitance of the capacitor. The product of $R$ and $C$ is called the time-constant $\tau = RC$. A capacitor is almost fully charged or fully discharged after about a time span of $4RC$.

Example: Suppose we would like a 10 µF capacitor to be almost fully charged in 0.1 second. Determine the value of the resistance that we need to place in series with this capacitor when connected to a battery. $4RC = 0.1$ which yields $R = 2.5$ kΩ.

Consider a resistor $R$ in series with a capacitor $C$ connected to a power supply as shown in the figure below. When the switch is closed (position 1), the capacitor begins to charge. The rate of charge of the capacitor depends on the circuit's RC time constant. Clearly a large resistance in the circuit limits the current, which slows down the charging of the capacitor. At the same time a capacitor with a large capacitance takes longer to fully charge.
After the capacitor is fully charged, when the switch is opened (position 2), the capacitor starts to discharge and the rate of discharge will depend on the value of the RC time constant. The significance of the value of RC can be seen by analyzing the charging or discharging equations for voltage or charge of the capacitor.

When the capacitor is fully charged, its voltage is $V_C = V_0$, the same as the voltage of the battery. As soon as the switch is thrown into position 2 (disconnected from the battery), the fully charged capacitor begins to discharge through the resistor. The capacitor voltage $V_C$ drops exponentially with time:

$$V_C = V_0 e^{-t/RC}$$

This equation indicates that the larger the RC, the longer it takes for the capacitor to discharge. For $t = RC$, $V_C = V_0/e$, or the capacitor voltage drops to $1/e$ of its maximum value ($V_C = 0.368 \times V_0$). For $t = 2RC$, the voltage drops to $V = 0.135 \times V_0$, etc… Therefore RC is a measure of the circuit's charging/discharging characteristics.

One can measure the time that it takes for the capacitors voltage to drop to 0.37 of its fully charged value and compare with the RC time constant of the circuit below.
In order to do so, one can monitor the voltage drop across the capacitor on an oscilloscope and carefully analyze the waveform.

**Measurement 1**

Capacitance measurement: Some multimeters are equipped for capacitance measurement. You can also use the large HP instrument in the back of the lab to determine the capacitance of your capacitors. You can also observe the charging and discharging of capacitors on an oscilloscope and estimate the value of capacitance through measurement of the RC time constant. In this part of the experiment we will try both methods.

A. Select a capacitor with a capacitance of approximately 50 nF. Using the HP capacitance meter measure its value. Pick a 2 kΩ resistor. Measure its resistance with a multimeter. Calculate the RC time-constant of the capacitor and resistor and record all values in your lab book.

B. Place the capacitor in series with the resistor and connect to a function generator. Select a square wave of 50% duty cycle. Set the frequency of the function generator to 4 kHz
and its amplitude to 2 $V_{pp}$ (or 1 $V_{p}$). Using the oscilloscope, observe the waveform of the function generator on channel 2 and the voltage across the capacitor on channel 1. The waveform on channel 1 should be similar to the charging and discharging curve shown above. Tweak the oscilloscope (and the frequency of the function generator) so you can observe one full charging and discharging cycle of the capacitor on the screen.

Using the oscilloscope screen divisions or cursors determine the time it takes for the capacitor to be fully discharged. Set that time equal to $4RC$ and by inserting the measured value of the resistor, calculate the value of $C$. Compare this capacitance with the value measured by the HP capacitance meter and record your findings in a table.

You can use your Discovery Scope to make the measurements at home. Indicate in your lab book which instrument you used.

**Measurement 2**

Series and parallel combination of capacitors: Similar to resistors, two or more capacitors can be connected in series or parallel. The formula for calculation of the equivalent capacitance of the combined capacitors is the reverse of resistors:

Series combination: $1/C = 1/C_1 + 1/C_2 + \ldots$

Parallel combination: $C = C_1 + C_2 + \ldots$

Therefore, if you wish to obtain a smaller capacitance, you can combine two or more capacitors in series and if you would like a larger capacitance, you can connect them in parallel.

Choose two capacitors with capacitances of about 100 nF and 200 nF (if unavailable, choose other values). Carefully measure each capacitance and record in your lab book.

A. Combine the two capacitors in series and using the HP capacitance meter measure the equivalent capacitance of the combination.

B. Combine them in parallel and measure the equivalent capacitance and record all values in your lab book.

C. Calculate the expected equivalent capacitance for each case and in a table compare with the measured values.
Measurement 3

Low-Pass and High-Pass Filters: In this part we will experiment two very simple filters consisting of a resistor and a capacitor. The idea is to demonstrate that we can filter out high frequency AC signals in Low-Pass filters and filter out the low frequency AC signals in High-Pass filters. The underlying principle of operation of RC filters is based on the fact that a capacitor exhibits a large electrical resistance at low frequencies, and a low electrical resistance at higher frequencies. Therefore we expect that a capacitor will act as an infinitely large resistor for DC signals and a very small resistance at very high frequencies. In a low-pass filter, the circuit blocks higher frequencies and in a high-pass filter, lower frequencies are blocked. Each filter has a threshold that is determined by the values of the resistance and capacitance of the two elements in the circuit. Therefore the threshold of the filters can be adjusted by changing the values of the resistance and capacitance of the elements in the circuit.

For both types of filters we apply a sine wave (input) to a circuit consisting of a series combination of a resistor and a capacitor. In a low-pass filter the output voltage is taken across the capacitor and in a high-pass filter the output voltage is taken across the resistor. In both cases the input and output voltages must share the ground connection. The cut-off frequency for these filters is defined to be $1/2\pi RC$, which corresponds to the frequency at which the output power is $1/2$ of the input power.

A. Low-pass filter

Connect the same circuit as measurement 1 above. Apply a sine wave with 4Vpp and 0V offset. You will be using both channels of the oscilloscope. Connect channel 2 to the input (function generator) and Channel 1 to the output (across the capacitor).

First, visually observe that at low frequencies the output has the same $V_{pp}$ as the input signal (low pass!). Gradually increase the frequency until you see a drop in the output $V_{pp}$. Now we need to define the usefulness range of this filter. This cut-off frequency is defined to be a frequency at which the power of the output signal drops to $1/2$ of the power of the input signal. At the end of this exercise we will show that at this point, which is called the -3 dB point, the amplitude of the output voltage is 0.707 of the input voltage. Therefore the objective of this part of the experiment is to identify the cut-off (or -3 dB) frequency. The -3 dB concept is quite fundamental in electronics but it takes some time for students to fully comprehend the meaning of it. Make sure that you ask your instructor if you have difficulty understanding this concept.
Set the cursors to measure and record the peak-to-peak output voltage when the output starts to drop. Choose an input P-P voltage of 5 V. Keep increasing the frequency until you observe that the P-P voltage $V_{out} = 0.707 V_{in}$. Make a note of this frequency.

**B. High-pass filter**

This experiment is identical to the previous part except, the output voltage is taken across the resistor and we start at high frequencies (high pass). At high frequencies the amplitude of the input and output signals are the same. Increase the frequency of the function generator until you see the height of the input and output signals are the same. Now reduce the frequency until the -3dB point is reached ($V_{out} = 0.707 V_{in}$). Make a note of this cut-off frequency. The circuit and the set up are shown below.

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Optional: Definitions of Cut-off and -3 dB frequencies

The cut-off threshold for the high-pass and low-pass filters is defined to be the frequency at which the output voltage drops to 0.707 of the input voltage. In general, the cut-off frequency is defined to be the frequency at which the output power drops to $\frac{1}{2}$ of the input power. However, since the electrical power dissipated in a component is proportional to $V^2$, then we conclude that the cut-off frequency corresponds to the location where the voltage drops to $(1/2)^{1/2}$ of its maximum, which is 0.707. Note that this point is also called the 3dB point. In a logarithmic scale the ration of the output to the input power is measured in a deciBel scale (or dB scale) is based on the following definition:

$$dB = 10 \log \left( \frac{P_{out}}{P_{in}} \right)$$

At the location where $P_{out} = (1/2) P_{in}$, then $dB = 10 \log(1/2)$, which leads to -3 dB. The negative sign signifies attenuation of the signal.