Electromagnetism: Introduction to the Course

Action-at-a-distance: Through Field

Example 1: Gravitational Field and Force

\[ F = -\frac{G m_1 m_2}{R^2} \hat{R}, \quad G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \]

Example 2: Electric Field and Force

Assume \( q_1 \) to be positive

\[ F_{q_1 q_2} = \left( \frac{1}{4\pi \varepsilon_0} \right) \frac{q_1 q_2}{R^2} (-\hat{R}) \]

\[ F_{q_1 q_3} = \left( \frac{1}{4\pi \varepsilon_0} \right) \frac{q_1 q_3}{R^1} \hat{R} \]

\( \varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m} \), \( \frac{1}{4\pi \varepsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2} \)

Numerical Examples

1. \( m_1 = m_2 = 1 \text{ Kg}, \ R = 1 \text{ m} \Rightarrow F = (6.67 \times 10^{-11}) \frac{1 \times 1}{1^2} \]

\[ F_{12} = 6.67 \times 10^{-11} \text{ N} \approx 67 \text{ pN} \]

2. \( q_1 = q_2 = 1 \text{ Coulomb}, \ R = 1 \text{ m} \Rightarrow F_{12} = (9 \times 10^9) \left( \frac{1 \times 1}{1^2} \right) \]

\[ \Rightarrow F_{12} = 9 \times 10^9 \text{ N} \]

See Appendix B for values of dielectric constant \( \varepsilon_r \) for various materials.

Examples: Teflon, paper, glass, Mica, Plexiglass, Distilled water
Electric Field (Units: \( \text{V/m} \))

\[
\vec{E} = \left( \frac{1}{4\pi \epsilon_0} \right) \frac{q}{R^2} \hat{R}
\]

Place a charge \( q' \) at a point \( P \) : it will experience a force:

\[
\vec{F}_{q'} = q' \vec{E}
\]

Force on \( q' \) — field of charge \( q \)

\[
\vec{E}_{q'} = \left( \frac{1}{4\pi \epsilon_0} \right) \frac{q q'}{R^2} \hat{R}
\]

The field \( \vec{E} \) defined above is in free space. If the charge is inside a material, then the electric field surrounding it will be reduced:

\[
\epsilon_{\text{const}} \vec{E} = \left( \frac{1}{4\pi \epsilon} \right) \frac{q}{R^2} \hat{R}
\]

where \( \epsilon = \epsilon_r \epsilon_0 \) where, \( \epsilon_r > 1 \)

Note: Units of \( \epsilon \):

\[
\text{Farad} \quad \text{F/m} \quad \text{Farad}
\]

\( \epsilon_r \) is dimensionless because

\[
\epsilon_r = \frac{\epsilon}{\epsilon_0}
\]

Inside materials it is convenient to define another quantity \( \vec{D} \) which is related to \( \vec{E} \):

\[
\vec{D} = \epsilon \vec{E}
\]

Electric flux density

Units of \( \vec{D} \):

\[
\left( \frac{\text{F}}{\text{m}} \right) \left( \frac{\text{m}}{\text{m}} \right) = \text{Coulomb} \quad \text{F} \quad \text{Volt}
\]

Farad is unit of capacitance

\[
Q = CV \quad \Rightarrow F = \frac{C}{V}
\]

Units of \( \vec{D} \):

\[
\frac{\text{C}}{\text{V} \cdot \text{m} \cdot \text{m}} \quad \Rightarrow \left( \frac{\text{C}}{\text{m}^2} \right)
\]

\[
\vec{D} = \epsilon \vec{E}
\]

\[
\frac{\text{C}}{\text{m}^2} \quad \text{F/m} \quad \text{V/m}
\]
Magnetic Field
Source of electric field: electric charges
Source of magnetic field: electric current element
\[ \vec{B} = \mu \frac{I}{r} \hat{\phi} \]
\[ \mu = \frac{\mu_0}{\mu_r} \]
\[ \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \] (Henry, unit of inductance)

Electrostatics:
Stationary charges (\( \frac{\partial q}{\partial t} = 0 \)) \( \vec{D} = \varepsilon \vec{E} \)

Magnetostatics:
Steady currents (\( \frac{\partial I}{\partial t} = 0 \)) \( \vec{B} = \mu_0 \vec{H} \)

Electrodynamics:
Time-varying currents (\( \frac{\partial I}{\partial t} \neq 0 \))
\( \vec{E} \) and \( \vec{B} \) are coupled to \( \vec{E} \) and \( \vec{H} \)

Electromagnetic Waves
\[ q \rightarrow \vec{E} \]
Stationary \( \rightarrow \vec{E} \) is constant at a given point

\[ q \rightarrow \vec{E} \]
Moving charge \( \rightarrow \vec{E} \) varies at a point

\[ q \rightarrow \vec{E}, \vec{v} \]
Accelerating charge \( \rightarrow \) energy is radiated

\[ q \rightarrow \vec{E} \]
Oscillating charge \( \rightarrow \) generates electromagnetic wave
As we said, electric current is the source of magnetic field. We know that the source of an electric current is moving charges. Moving charge $\rightarrow$ electric current $\rightarrow$ magnetic field.

The source of electromagnetic waves/radiation is moving/accelerating electric charges.

Waves are defined by their wavelength $\lambda$ and frequency $f$:

$$\lambda \cdot f = c$$

Waves, velocity $\rightarrow$ wavelength $\rightarrow$ frequency

- $c = 3 \times 10^8 \text{ m/s}$ (speed of light)

<table>
<thead>
<tr>
<th>Type</th>
<th>Frequency</th>
<th>Wavelength</th>
<th>Main uses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y-rays</td>
<td>$10^{18}$</td>
<td>1 Å</td>
<td>Communication, radar, radio astronomy</td>
</tr>
<tr>
<td>X-rays</td>
<td>$10^{16}$</td>
<td>1 nm</td>
<td></td>
</tr>
<tr>
<td>UV</td>
<td>$10^{15}$</td>
<td>1 μm</td>
<td>Communication, radar, radio astronomy</td>
</tr>
<tr>
<td>IR</td>
<td>$10^{14}$</td>
<td>1 mm</td>
<td></td>
</tr>
<tr>
<td>Radio</td>
<td>$10^{11}$</td>
<td>1 m</td>
<td>Communication, radar, radio astronomy</td>
</tr>
<tr>
<td>Optical</td>
<td>$10^{10}$</td>
<td>10 m</td>
<td></td>
</tr>
</tbody>
</table>

$\rightarrow$ E&M waves travel through:

- Vacuum/air
- Transmission Line
- Waveguide
- Optical fibers

$\rightarrow$ E&M waves are generated by:

- Transmitters/Antennas

$\rightarrow$ E&M waves are received by:

- Antennas
Wave Equation

→ Take a picture of an oscillating string (freeze time)

\[ y(x) = A \cos \left( \frac{2\pi}{\lambda} x \right) \]
\[ \lambda = \text{wavelength} \]

→ Now observe the up and down motion of one point on the string (for fixed x).

\[ y(t) = A \cos \left( \frac{2\pi}{T} t \right) \]
\[ T = \text{Period} \]

Plot \( y(t) \) vs. \( t \) →

Combine \( y(x) \) and \( y(t) \): both \( x \) and \( t \) are variable

\[ y(x, t) = A \cos \left( \frac{2\pi}{T} t - \frac{2\pi}{\lambda} x \right) \]

\( \sqrt{\text{negative sign means that}} \)
\[ \text{The wave is traveling in } +x \text{ direction} \]

\[ \text{Note: } \quad f = \frac{1}{T} \]

\[ \Rightarrow y(x, t) = A \cos \left( 2\pi f t - \frac{2\pi}{\lambda} x \right) \]

Also:
\[ \omega = 2\pi f \quad \text{and} \quad \beta = \frac{2\pi}{\lambda} \]

\[ \text{Angular frequency} \]
\[ \text{Wave number} \]

\[ y(x, t) = A \cos (\omega t - \beta x) \]

\[ \text{Propagation velocity: } U = f \lambda = \frac{\lambda}{T} \text{ or, } \omega = U \beta \]
Phasor representation of a traveling wave:

\[ Y(x,t) = A e^{j(wt - \beta x)} \]  \hspace{1cm} (1)

**Note:**
\[ e^{j\theta} = \cos \theta + j \sin \theta \]  \hspace{1cm} (2)

The real part of (1) according to (2):

\[ Y(x,t) = A \cos(wt - \beta x) \]  \hspace{1cm} \text{(Real)}

Wave traveling through a lossy medium:

\[ Y(x,t) = A e^{-(\alpha x + j(wt - \beta x + \phi))} \]

- Absorption constant
- Phase constant
- Decaying envelope
- General representation of a wave

Rectangular and polar representation of complex quantities:

\[ Z = X + jY = |Z| e^{j\theta} \rightarrow |Z| [\cos \theta + j \sin \theta] \]

\[
\begin{align*}
X &= |Z| \cos \theta \\
Y &= |Z| \sin \theta
\end{align*}
\]

\[ \theta = \tan^{-1} \left( \frac{Y}{X} \right) \]

Complex conjugate: \( Z^* = X - jY \)

\[ |Z| = \sqrt{Z Z^*} = \sqrt{X^2 + Y^2} \]

Additional, subtraction, multiplication and division of complex quantities.
Example 1-3 (page 35)

\[ V = 3 - j4 \quad \text{and} \quad I = -(2 + j3) \]

(a) Express \( V \) and \( I \) in polar form

\[ |V| = \sqrt{9 + 16} = 5 \quad , \quad |I| = \sqrt{4 + 9} = \sqrt{13} = 3.61 \]

\[ \Theta_V = \tan^{-1} (-\frac{4}{3}) \quad \Theta_I = 180^\circ + \tan^{-1} (\frac{3}{2}) \]

\[ \Rightarrow \Theta_V = -53.1^\circ \quad \Theta_I = 180^\circ + 56.3^\circ \]

\[ V = |V| e^{j\Theta_V} \quad I = |I| e^{j\Theta_I} \]

\[ V = 5 e^{-j53.1^\circ} \quad I = 3.61 e^{j236.3^\circ} \]

or,

\[ V = 5 \angle -53.1^\circ \quad I = 3.61 \angle 236.3^\circ \]

(b) Calculate \( VI \)

\[ VI = (5)(3.61)e^{j(236.3 - 53.1)} \]

\[ \Rightarrow VI = 18 e^{j183.2^\circ} \]

(c) \( VI^* \)

\[ I^* = 3.61 e^{-j236.3^\circ} \]

\[ VI^* = 18 e^{-j289.4^\circ} \]

\(-289.4^\circ \) is the same as \(+70.6^\circ \)

\[ \Rightarrow VI^* = 18 e^{j70.6^\circ} \]

(d) \( \frac{V}{I} \)

\[ \frac{V}{I} = \frac{5 e^{-j53.1^\circ}}{3.61 e^{236.3^\circ}} \]

\[ \Rightarrow \frac{V}{I} = 1.39 e^{-j289.4^\circ} \]

\[ \frac{V}{I} = 1.39 e^{j70.6^\circ} \]

(e) \( \sqrt{I} \)

\[ \sqrt{I} = (3.61 e^{j236.3^\circ})^{\frac{1}{2}} = 1.9 e^{j118^\circ} \]

where, \( \sqrt{3.61} = \pm 1.9 \)
Review of Phasors

Many engineering problems are cast in the form of linear integro-differential equations. If the excitation (force function) is a harmonic function (sinusoidal function of time), we use phasor notation to convert the equations into linear equations with no sinusoidal functions. This technique will simplify solving the problem! After the solution is achieved, one converts it from the phasor domain back to the time domain.

In the event that the excitation is non-sinusoidal, one can first convert it to sine functions through Fourier series and then apply the phasor technique.

For non-periodic source functions (such as a single pulse), it can be expressed as a Fourier Integral and then converted to phasors.

Consider the following time-domain function:

\[ U_s(t) = V_0 \sin(\omega t + \phi_0) \]

Kirchhoff's voltage law:

\[ U_s(t) = U_R(t) + U_C(t) \]

This is a differential-integral eqn. in time-domain, which is cumbersome to solve.

To simplify the method of solving eqn. ①, let us represent the quantities in phasor forms:

\[ e^{j\omega t} = \cos \omega t + j \sin \omega t \implies \cos \omega t = \text{Re} [e^{j\omega t}] \]
First convert $U_s$ in sine form to cosine form ($\sin x = \cos \left( \frac{\pi}{2} - x \right)$)

$U_s = V_0 \sin (wt + \phi_0) = V_0 \cos \left( \frac{\pi}{2} - wt - \phi_0 \right)$

But $\cos (x) = \cos (-x) \Rightarrow U_s = V_0 \cos (wt + \phi_0 - \frac{\pi}{2})$

From the arguments given at the bottom of the previous page,

$U_s = \Re \left[ V_0 e^{j(\phi_0 - \frac{\pi}{2})} e^{jwt} \right] = \Re \left[ V_0 e^{j(\phi_0 - \frac{\pi}{2})} ^{\hat{U}}_s e^{jwt} \right]$

$\Rightarrow U_s = \Re \left[ ^{\hat{U}}_s e^{jwt} \right]$, where $^{\hat{U}}_s = V_0 e^{j(\phi_0 - \frac{\pi}{2})}$

We can write a similar eqn. for $I(t)$ of the R-C circuit:

$I(t) = \Re \left[ ^{\hat{I}}_c e^{jwt} \right]$

It is now easy to take the time derivative or integral of $I(t)$.

$\frac{d}{dt} \left[ \Re \left( \frac{^{\hat{I}}_c}{jw} \right) e^{jwt} \right]$

Rewrite eqn. of the previous page using phasors:

$U_s(t) = R \hat{I} + \frac{1}{C} \int \Re \left( \frac{^{\hat{I}}_c}{jw} \right) e^{jwt} dt$

$\Rightarrow \Re \left\{ \left[ ^{\hat{U}}_s - (R + \frac{1}{jwC}) \hat{I} \right] e^{jwt} \right\} = 0$

$\Rightarrow ^{\hat{U}}_s = \left( R + \frac{1}{jwC} \right) \hat{I} \Rightarrow \hat{I} = \frac{^{\hat{U}}_s}{R + \frac{1}{jwC}}$

Now, rewrite the expression for $I(t)$ from (3) and substitute (2) and (4):

$I(t) = \Re \left[ ^{\hat{I}}_c e^{jwt} \right] = \Re \left[ \frac{^{\hat{U}}_s}{R + \frac{1}{jwC}} e^{jwt} \right] = \Re \left[ \frac{V_0 e^{j(\phi_0 - \frac{\pi}{2})}}{R + \frac{1}{jwC}} e^{jwt} \right]$

See text, p.41 for the details of how the above expression for $I(t)$ can be written as (by going from polar to rectangular coordinates):

$I(t) = \frac{V_0}{\sqrt{1 + w^2 R^2 C^2}} \cos (wt + \phi - \phi)$ where, $\phi = \tan^{-1} wC$