P-N Junction Capacitance

Consider P-N junctions in reverse-bias and forward-bias modes.

1. Reverse Bias:
   Separation of fixed charges in the depletion region creates a capacitor similar (not identical) to the simple parallel-plate capacitor.

2. Forward Bias:
   The excess electrons injected into the P-side, and the excess holes injected into the N-side create a separation of positive and negative charges, creating capacitive conditions.

Therefore, under AC conditions, the capacitance of the P-N junction under forward and reverse biases plays an important role.

Calculation of capacitance of a P-N junction:

In general, \( dQ = C \cdot dV \) for a capacitor. Of course, if \( V \) changes linearly with \( Q \), then

\[
C = \frac{Q}{V}.
\]

But \( Q(V) \) has a non-linear behaviour for P-N junctions.
Consider a p-n junction page 79 notes, at equilibrium:

\[ W = \left[ \frac{2eV_0}{q} \left( \frac{N_a + N_d}{N_aN_d} \right) \right]^{1/2} \]

Under applied bias \( V \):

\[ W = \left[ \frac{2e(V_0 - V)}{q} \left( \frac{N_a + N_d}{N_aN_d} \right) \right]^{1/2} \]

where \( \begin{cases} \text{Forward: } V > 0, \text{ positive} \\ \text{Reverse: } V < 0, \text{ negative} \end{cases} \)

we also saw that:

\[ X_{p0} = \frac{N_d}{N_a + N_d} W \quad \text{and} \quad X_{n0} = \frac{N_a}{N_a + N_d} W \]

Charge neutrality requires that \( Q_+ = Q_- \)

\[ |Q| = q(Volume)(N_a) = q(Volume)(N_d) \]

\[ \frac{X_{p0} A}{X_{n0} A} \]

Calculate the positive charge \( Q \) on the n-side:

\[ Q = qX_{n0} A N_d = qA \left( \frac{N_a}{N_a + N_d} \right) W N_d \]

\[ Q = qA \frac{N_a N_d}{N_a + N_d} W \quad \text{from (4)} \]

Substitute for \( W \) from (2) in (5):

\[ Q = qA \frac{N_a N_d}{N_a + N_d} \left[ \frac{2e(V_0 - V)}{q} \left( \frac{N_a + N_d}{N_aN_d} \right) \right]^{1/2} \]

Take the factor \( q \left( \frac{N_a N_d}{N_a + N_d} \right) \) inside the bracket:

\[ Q = A \left[ 2q(V_0 - V) \left( \frac{N_a N_d}{N_a + N_d} \right) \right]^{1/2} \]

Now, define the junction capacitance \( C_j = \left| \frac{dQ}{dV} \right| = \left| \frac{dQ}{d(V_0 - V)} \right| \)
Rewrite eqn. (6) of last page:

\[ Q = A \left[ 2 q \varepsilon (V_o - V) \left( \frac{N_a N_d}{N_a + N_d} \right) \right]^{1/2} \]

\[ Q = A \left[ 2 q \varepsilon \left( \frac{N_a N_d}{N_a + N_d} \right) \right]^{1/2} (V_o - V)^{1/2} \]

\[ C_j = \left| \frac{dQ}{d(V_o - V)} \right| = \frac{1}{2} A \left[ 2 q \varepsilon \left( \frac{N_a N_d}{N_a + N_d} \right) \right]^{1/2} (V_o - V)^{-1/2} \]

\[ \therefore \quad C_j = \frac{A}{2} \left[ \frac{2 q \varepsilon}{(V_o - V)} \left( \frac{N_a N_d}{N_a + N_d} \right) \right]^{1/2} \]  \hspace{1cm} (7)

Take the factor \( \frac{1}{2} \) inside the bracket and bring \( \varepsilon \) outside:

\[ C_j = \varepsilon A \left[ \frac{q}{2 \varepsilon (V_o - V)} \left( \frac{N_a N_d}{N_a + N_d} \right) \right]^{1/2} \] \hspace{1cm} (8)

\[ \text{same as} \quad \frac{1}{W} \quad \text{from eqn. (2) of page 101} \]

\[ \therefore \quad C_j = \frac{\varepsilon A}{W} \]

which is the equivalent of the capacitance for a parallel plate capacitor of cross sectional area \( A \) and plate separation of \( W \).

For a \( p^+ - n \) junction: \( N_a >> N_d \Rightarrow X_{no} >> X_{p0} \) according to (3) and (4) of page 101

Thus, the depletion width \( W \) is mostly on the less doped side of the junction (\( n \)-side):

\[ \frac{N_a N_d}{N_a + N_d} \quad \text{Na} >> \text{Nd} \quad \rightarrow \quad \frac{N_a N_d}{N_a} \quad \rightarrow \quad \text{Nd} \]

\[ \therefore \quad \text{eqn. (7) reduces to: } \quad C_j = \frac{A}{2} \left[ \frac{2 q \varepsilon}{(V_o - V)} N_d \right]^{1/2} \] \hspace{1cm} (9)

\[ \text{for a } p^+ - n \text{ junction} \]
A measurement of $C_j$ of a p-n junction under a bias of $V$ could lead to the determination of $N_d$, which is the doping concentration of the lightly doped side.

Rewrite eqn. 9 of the previous page:

$$C_j = \frac{A}{2} \left[ \frac{2q\phi}{(V_o-V)} N_d \right]^{1/2}$$

This equation predicts the junction capacitance of an abrupt p-n junction. The equation must be modified for graded junctions.

**Note:** The junction capacitance $C_j$ is dominant for reverse bias.

**Forward bias capacitance:**
Also called storage capacitance.

For forward bias, the diffusion capacitance will be dominant.

The diffusion capacitance ($C_s$) becomes negligible when the length of the sample is larger than the diffusion length ($L_p$) of the injected minority carriers. Without proof: The diffusion capacitance due to the stored holes on the n-side is given by:

$$C_s = \frac{1}{3} \frac{q^2}{kT} A L_p n^2 e^{qV/kT} \rightarrow \text{for short samples}\quad l \ll L_p$$

Examples of short diodes: Most silicon p-n junction.

Examples of long diodes: Direct band gap laser diodes.

It can be shown that the a-c conductance of the p-n junction would be:

$$G_s = \frac{dI}{dV} = \frac{q}{kT} I$$

**Example:** Find $C_j$ for $N_d = 10^{15} \text{cm}^{-3}$, $N_a = 10^{17} \text{cm}^{-3}$ in a silicon diode of $A = 6.4 \text{cm}^2$.

Use eqn. 7 of p. 102:

$$C_j = \frac{A}{2} \left[ \frac{2q\phi}{(V_o-V)} \left( \frac{N_a N_d}{N_a + N_d} \right) \right]^{1/2}$$

$$V_o = 4 \text{Volts}$$

$$\Rightarrow C_j = 4.8 \times 10^{-13} \text{ F} = 0.42 \text{ pF}$$

Text, p. 221
The Varactor Diode

Varactor \rightarrow \text{variable reactor, which is a voltage-controlled variable capacitance for reverse-bias voltages.}

Recall eqn. (7) of page 102: \[ C_j = \frac{A}{2} \left[ \frac{2 e^\frac{E}{kT}}{C (V_o - V)} \left( \frac{N_a N_d}{N_h + N_h} \right) \right]^{1/2} \]

In the event that a fixed capacitor is desired, then the reverse voltage \( V \) is kept constant.

\textbf{Note:} Traditional tuning circuits use variable capacitors. In order to reduce the size/weight, a varactor may be used instead.

Other applications:
- Harmonic generation
- Microwave frequency multiplication
- Active filters

As noted by the expression for \( C_j \) in eqn. (7), the capacitance varies with \( \frac{1}{(V_o - V)^2} \) for abrupt junctions. However, for graded junctions, \( C_j \) varies with \( V^{-n} \), \( V \gg V_0 \).

\( \circ \) Voltage sensitivity for \( C_j \) is higher for abrupt junctions, which means varactor diodes are fabricated using epitaxial growth which are suitable for creation of abrupt junctions.

Varactors in L-C circuits:

The resonant frequency of an L-C circuit consisting of an inductor and a varactor will be voltage-dependent:

\[ \omega_r = \frac{1}{\sqrt{L C_j}} = \frac{1}{\sqrt{(L)(\text{constant}) \cdot (V_o - V)^{1/2}} \left( V_{m} \right) \text{R} \ L \ C \ V_{\text{out}} \]

\( \circ \) \( \omega_r \) would be proportional to \( V_r \) assuming \( V_r \gg V_0 \).

\( \circ \) \( \omega_r \propto V_r \) \rightarrow \text{which indicates voltage-controlled resonance}
Metal-Semiconductor Junctions

The junction between metals and semiconductors may behave as:

a. Rectifying contact \( \rightarrow \) Similar to p-n junctions
   Also called Schottky barriers

b. Non-rectifying contact \( \rightarrow \) Ohmic contacts

The ohmic contacts are obviously of great importance when
leads are connected to semiconductor devices. On the other hand,
many properties of p-n junctions can be realized using metal-semiconductor
contacts which are simple to fabricate. In order to understand
how these junctions work, let us first consider the metal
work function \( \Phi_m \), which is the energy necessary to provide
to an electron in a metal to free it from the metal.

Free electrons can eject from a
clean metallic surface if an incident
beam of energy equal or exceeding \( \Phi_m \)
is used.

Typical values of \( \Phi_m \):
- Aluminum \( \rightarrow \) \( \Phi_m = 4.3 \text{ eV} \)
- Gold \( \rightarrow \) \( \Phi_m = 4.8 \text{ eV} \)

In order to understand the rectifying properties of metal-
semiconductor contacts (Schottky barriers), let us look at
the energy band diagram of metals:

At low temperatures, the Fermi energy \( E_F \)
indicates the highest energy level
of electrons. At \( E_F \), when
an electron is given \( \Phi_m \) energy,
it breaks free.
**Metal-Semiconductor Contacts:**

Now look at the energy band diagrams of a metal and a semiconductor separately side-by-side.

$q\Phi_m$ → energy needed to free an electron from the metal surface (metal work function)

$q\Phi_S$ → Semiconductor work function

$q\phi_X$ → electron affinity

In the picture on the right, the semiconductor Fermi energy $E_{FS}$ is higher than the metal Fermi energy. When the two sides come into contact, a process similar to a p-n junction will occur.

The energy band of the semiconductor must lower itself so its Fermi energy $E_{FS}$ can line up with the metal Fermi energy $E_{FM}$. Recall that the Fermi level for devices at equilibrium must be flat!

When $E_C$ and $E_V$ are lowered, $E_{FS}$ lines up with $E_{FM}$. This is actually done by the migration of electrons in the valence band to the metal. These electrons on the semiconductor side possess higher energies than the neighboring electrons in the metal. As this migration proceeds, the migrating electrons leave behind positive space charges. As a result, an electric field is set up. This electric field grows stronger as more electrons migrate from right to left. Eventually, the strength of the electric field prevents any further electron migration from the n-type semiconductor to the metal.
Metal-Semiconductor Contact Energy Band Diagram:

a. N-type:
The migration of electrons gives rise to an electric field which opposes further migration of electrons to the left.

The band diagram slopes up as we move from right to left.

This is consistent with the direction of the electric field $E$.

---

b. P-type:
In this case, the Fermi energy of the metal is higher than the Fermi energy of the p-type semiconductor before contact. Immediately after contact is made, the two Fermi energy levels shift to line up and create equilibrium. As a result, the semiconductor $E_c$ and $E_V$ lines must move up to raise the lower $E_F$ high enough to line up with $E_F$.

Clearly, in this case, the electric field established in the junction points from left to right. The reason is that the electrons leaving the metal near the junction leave a positive space charge behind, and the migration makes the right-hand side of the junction positive.
Rectification of metal-semiconductor contacts

Consider the contact between a metal and an n-type semiconductor.

**Equilibrium**

**Forward Bias** $V_f$

**Reverse Bias** $V_r$

In forward bias, the depletion region width is reduced from $W_0 \rightarrow W_r$.

And in reverse bias, the depletion region width is increased $W_r \rightarrow W_0$.

The great similarity between the Schottky junctions and the $p-n$ junctions make us conclude that the I-V characteristics of these two types of junctions is similar.

$$I = I_0 \left( e^{\frac{qV}{kT}} - 1 \right)$$

For both n-type and p-type metal-semiconductor contacts.

**Note:** Forward current is due to the injection of majority carriers (electrons) from the semiconductor side to the metal side. The absence of minority carrier injection is an interesting feature of these devices → little storage delay times.
Ohmic Contacts

In many cases we do not wish for a rectifying metal-semiconductor contacts. In a typical integrated circuit, many p-n junctions are to be connected to each other and other devices using metal wires. At these points of contact a non-rectifying Ohmic contact is desired. The idea behind an ohmic contact is to minimize the contact potential.

A practical method for forming ohmic contacts is by doping the semiconductor heavily in the contact region. The result of this heavy doping is to reduce the depletion width. Even if a small contact potential barrier $V_0$ exists, because it is very narrow, electrons from the metal side can tunnel through the barrier and as a result the Fermi levels of both sides line up and remove the rectifying circumstances.

**Example:** Create an $n^+$ layer in the silicon side of the $n$-type Si contact by heavily doping it with $Al$ (containing a small percentage of $Sb$). This results in an excellent Ohmic contact. One can then connect $Al$ wire to it.

**Example:** Similarly, to make an Ohmic contact on a p-type Si, we need to heavily dope it and create a $p^+$ layer. Aluminum is a good acceptor level for Si. Therefore one can thermally anneal $Al$ on the p-type Si before connecting an Al wire to it.

**Note** In a p-n junction -> the region surrounding the junction area is in the middle of the crystal structure, void of imperfections. However, for a metal-Semiconductor contact, the metal is deposited on the surface of the semiconductor where the crystal periodicity has been disrupted. This creates unwanted interface states that could act as traps for carriers.
Heterojunctions

Up to this point, we have discussed P-N junctions and metal-semiconductor junctions. Another important class of junctions is between two different semiconductors → heterojunctions.

Applications: Bipolar transistors, field-effect transistors, and semiconductor lasers.

When two dissimilar semiconductors are brought together to form a junction, in order for the Fermi energy level to be uniform across the junction, the energy bands must undergo discontinuities.

\[ \begin{align*}
E_{c_1} & \quad \text{P-type} \\
E_F & \\
E_{v_1} & \\
E_{c_2} & \quad \text{n-type} \\
E_F & \\
E_{v_2} &
\end{align*} \]

When a junction is formed, \( E_{F_2} \) must be lowered to align itself with \( E_{F_1} \).

Curvatures (electric field E):

1. Same as \( E_c \)
2. Curvature(1)
3. Same as \( E_v \)
4. Curvature(4)

See text, page 239 for a more thorough analysis of the band diagram of this hetero P-N junction. However, a crude way of explaining the discontinuity involves the necessity for the curvature of \( E_c \) and \( E_v \) to be identical for each side of the junction. Therefore, we could not have a heterojunction band diagram such as:

Because the band curvature (electric field E) is not the same everywhere!!
Note that for heterojunctions, the boundary conditions at the junction require that: \[ E_1 E_1 = E_2 E_2 \]

\( \text{electric field} \)

We can use similar equations to homojunction equations for calculation of built-in potential and depletion region width. However, we must take into account the different permittivities of the two sides in heterojunctions.

An example of a heterojunction: \( \text{GaAs - AlGaAs} \)

\[ E_g = 1.43 \text{ ev} \quad \text{and} \quad E_g \text{ depends on} \]

The composition of Al and Ga

For example, for \( \text{Al}_{0.3} \text{Ga}_{0.7} \text{As} \):

\[ E_g = 1.85 \text{ ev} \]

Whereas for other compositions \( E_g \) would be different.

A two junction \( \text{Al}_{0.3} \text{Ga}_{0.7} \text{As} - \text{GaAs} - \text{Al}_{0.3} \text{Ga}_{0.7} \text{As} \)

would form a single quantum well device for LED and Laser diode applications (see text, p. 88, Fig 3-13).

Later we will see how to engineer the composition's band gap by varying \( x \):

\( \text{Al}_x \text{Ga}_{1-x} \text{As} \).
Semiconductor Optoelectronic Devices

In the previous part we discussed electronic properties of diodes specifically for electronic applications: Rectifying diodes, switching diodes, Zener and Avalanche diodes, Varactors, etc...

In this part, we will examine optical properties of diodes specifically for optoelectronic systems involving light emitters and detectors.

As an example, consider an optical fiber link for transmitting digital information:

[Diagram of optical fiber link]

In a more complete optoelectronic system, one must include:

- Wavelength division multiplexer, optical amplifier, Add-drop multiplexer, Wavelength division demultiplexer, \( \lambda_i \)-pass filter, Analog receiver, and output signal conditioning.

[Diagram of complete optoelectronic system]

In this course we will focus on the electronic properties of light emitters (LEDs and Laser diodes), and detectors. The rest of the optoelectronic system will be discussed in the Optical Fiber Communication Course. First we will discuss light sources!
Light Emitting Diodes (LEDs)

Earlier when discussing diodes with electronic applications, most often we looked at recombination of electrons and holes as undesired events (with some exceptions including fast switching diodes). Nevertheless, we did not look at the outcome of such recombination events. However, in the case of LEDs, we are specifically interested in the emission of light following such recombinations. For efficient light generation, transfer of electron energies to lattice vibrations must be avoided. It is therefore necessary that we use direct band gap semiconducting materials such as GaAs for LEDs. However, indirect bandgap materials such as GaP are commonly used for light emission, provided they are doped with appropriate impurities that create an environment similar to direct band gap transitions.

![Diagrams of indirect and direct band gaps](image)

Example Si:

Here, the gap for the transition results in a significant lattice vibrations.

Example GaAs:

Example GaP: N

Nitrogen doped GaP

The energy level of Nitrogen is about 0.1-0.2 eV below the conduction band. Although some lattice vibration is generated when the impurity center traps the electron, the transition from the nitrogen levels to the V-band is quite efficient and emits green LED light. Since hf < Eg, the emitted light is not absorbed by Ga.
Homojunction LEDs

An LED is essentially a p-n junction diode typically made from a direct band gap semiconductor, or a properly doped indirect band gap material as discussed on the last page. The LED emits light in forward bias conditions:

At equilibrium, the built-in potential/electric field prevents transitions of electrons from the heavily doped n region to the p-region.

Similar to the case of electronic diodes:

\[ qV_0 = kT \ln \left( \frac{N_h N_n}{n_i^2} \right) \]

and

\[ W = \left( \frac{2eV_0}{q} \left( \frac{1}{N_h} + \frac{1}{N_n} \right) \right)^{1/2} \]

A p-n⁺ LED is preferred over a p⁺-n LED because the electrons generally possess a higher mobility than electrons.

Now, apply a forward bias \( V_F \):

The electrons from the n⁺-side are injected into the p-side, where they recombine with holes, resulting in a spontaneous emission of photons. Recombination primarily occur within the depletion region and
Within a volume extending over one diffusion length of electrons ($L_n$) in the p-side of the junction.

**Spontaneous Emission of Radiation:**

This type of light emission is the result of the statistical nature of the natural recombination process of electrons and holes. The photons are generally emitted in random directions.

**Note:** It is important the emitted light is not reabsorbed by the LED to cause transition of electrons from the V-band back to the C-band.

→ One way to deal with this problem is to make the p-side thin enough, so the emitted light has less of a chance of absorption.

→ Alternatively, one can fabricate a heterostructure LED. This topic will be discussed in the next section.

→ Also note that in forward bias, some holes will be injected from the p-side into the $N^+$ side. However, the number of injected holes will be much smaller than the injected electrons.

→ Furthermore, note that the resulting current in a reverse bias due to diffusion and thermal generation of E-H pairs is very small and does not lead into any significant emission of light. The reverse saturation current is similar to the case of electronic diodes and is given by:

$$I_0 = QA \left( \frac{D_p}{E_p} P_n + \frac{D_n}{E_n} N_p \right)$$

Obviously, $P_n << N_p$ for a $p-N^+$ junction.
Heterojunction LEDs

As we mentioned earlier, one way to remove the problem of reabsorption of the emitted light in an LED would be to make the material on one side of the junction of a wider band gap material compared to the other side \((E_g_2 > E_g_1)\). Assuming that the light is generated on the side of the junction with a smaller \(E_g\), then the energy of the emitted light \(h\nu\) will be smaller than \(E_g_2\): \(h\nu < E_g_2\), which means that the light energy cannot induce band-to-band transitions on the wide band gap side.

Earlier we mentioned that another alternative would be to make the P-side of a homojunction LED thin so the light has little chance of absorption before escaping the diode. However, this alternative has an inherent drawback: A thin P-layer means the surface defects due to the disruption of the periodic structure of the P-side are within the reach of the electrons injected from the N\(^+\) side (within one diffusion length \(L_n\)). This results in trapping of electrons by these defects, which reduces the efficiency of light emission.

Note: A wider band gap material has a lower index of refraction, which would result in total internal reflection of the light on the wider band gap side. As a result the emitted light would bounce back and forth along
The wide $E_g$ side providing it a chance to exit the LED.

$\theta_2 > \theta_1$, since $n_1 > n_2$

Critical angle: $n_1 \sin \theta_c = n_2 \sin 90^\circ \Rightarrow \sin \theta_c = \frac{n_2}{n_1}$

Total Internal Reflection (TIR) occurs when $\theta_1 > \theta_c$.

Depending on the physical structure of the LEDs, some are Surface Emitting while others are Edge Emitting LEDs.

**Note 1:** A heterostructure could include more than one heterojunction.

**Note 2:** Electron and hole concentration convention in heterojunctions:

Small band gap side: $\{N_\text{p}\}$

Wider band gap side: $\{N_\text{p}\}$

Next, we will study heterojunctions in some detail.
Alignment of Energy bands in Heterostructures

**Type I:**

This type of band alignment is applicable to:
- GaAs
- AlGaAs
- Ga_x In_{1-x} As
- InP
- Ga_x In_{1-x} P

**Type II:**

This type of band alignment is applicable to the junction between:
- Ga_x In_{1-x} As
- GaAs_y Sb_{1-y}
- Ga_x In_{1-x} As
- GaAs_y Sb_{1-y}

**Type I and II distinctions are based on the relative magnitudes of $\chi_1$ and $\chi_2$.**

**Type I junctions are more common among optoelectronic devices.**

Next, we will consider the following heterojunctions:
- NP, PP, N^+P P: Recall that N and P are assigned to smaller band gap material, and N^+ and P^+ are for wider band gap material
(a) NP Heterojunction

At equilibrium:

$E_F$ is uniform through the junction.

(b) PP Heterojunction

The band edge profiles for a given heterojunction are determined by the doping levels, carrier transport, and recombination in the junction area.

This is similar to homojunctions.

(c) NPP → a double heterojunction

This double heterojunction is obtained by combining NP and PP junctions described above:

\[
\begin{array}{c|c|c}
N^+ & p & P \\
AlGaAs & GaAs & AlGaAs \\
\end{array}
\]

Equilibrium:

Biased:

Now, apply a forward bias applied to the PP junction.

Electrons in the $N^+$ region are injected, and electrons confined in the $P$ region recombine with holes and produce light.

The light goes through the wide bandgap $N^+$ region and exists the device.
LED Output Spectrum

The output power/intensity of an LED versus the wavelength of the emitted light forms a spectrum which is sharper at lower temperatures and broader at higher temperatures.

For GaAs:

Note: The peak of the spectrum shifts to the right with temperature increase.

$\Delta \lambda \rightarrow$ represents the sharpness of the distribution: Width at half height

$E_g = 1.43 \text{ ev for GaAs}$

The wavelength corresponding to $E_g$:

$$\lambda = \frac{1240 \text{ nm ev}}{E_g}$$

$\Rightarrow \lambda = 867 \text{ nm}$

But, the experimental results shown above indicate that the peak of radiation at room temperature (25°C) is 822 nm. Why is the peak not at 867 nm?

Find the energy corresponding to 822 nm: $E = \frac{1240 \text{ nm ev}}{822} = 1.51 \text{ ev}$

Find the difference: $1.51 \text{ ev} - 1.43 \text{ ev} = 0.078 \text{ ev}$

So we conclude that the probability of transition is maximum for electrons that have about 0.039 ev energy above the C-band.

$\frac{0.039}{0.026} = 1.5$
We conclude that the transition probability is the highest for electrons that reside in energy levels above the conduction band edge by \( 1.5 \, kT \) energy.

This observation is in full agreement that the density of states for electrons is the highest at an energy slightly above the C-band. Similarly, the density of states for holes is maximum at energies slightly below the V-band.

For GaAs: experimental values for light output of LED

<table>
<thead>
<tr>
<th>( T ) (°C)</th>
<th>( \lambda_0 ) (nm)</th>
<th>( \Delta \lambda ) (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>804</td>
<td>30</td>
</tr>
<tr>
<td>25</td>
<td>822</td>
<td>40</td>
</tr>
<tr>
<td>85</td>
<td>837</td>
<td>48</td>
</tr>
</tbody>
</table>

The larger \( \Delta \lambda \) at higher temperatures indicate that the spectrum widens at higher temperatures. The peak wavelength and the line width of the beam are important parameters of an LED, which could play a crucial role if used for communication purposes.
**Bandgap Engineering**

The light emitted by a diode laser or LED follows the relationship for wave velocity:

\[ c = f \lambda \quad (1) \]

This light is radiated because an electron lowers its energy by an amount \( E \rightarrow h \frac{c}{\lambda} \quad (2) \)

\[ E \approx \frac{1240 \text{ (eV.nm)}}{\lambda \text{ (nm)}} \quad (3) \]

Eqn. (3) connects the wavelength of emitted light to the electron energy loss \( E \). This simple equation is quite useful for calculation of the necessary band gap for certain light wavelengths.

**Example 1:** What energy band gap would result in radiation of light of \( \lambda = 1310 \text{ nm} \)

\[ E = \frac{1240}{1310} \approx 0.95 \text{ eV} \]

**Example 2:** Find \( E \) for \( \lambda = 1550 \text{ nm} \)

\[ E = \frac{1240}{1550} \Rightarrow E = 0.8 \text{ eV} \]

One way to create the needed \( E_g \) would be to make alloys of different materials. However, it is absolutely necessary for the constituents of the alloy to have very similar lattice spacings. Otherwise, any significant lattice mismatch will create major defect concentrations that will radically reduce the device efficiency.
Example: What $E_g$ will result in the emission of blue light? Assume $\lambda = 4.50\,\text{nm}$.

$$E_g = \frac{1240}{450} \implies E_g \approx 2.75\,\text{eV}$$

The chart above describes the energy gaps of many binary semiconductors on the vertical axis and their lattice spacing on the horizontal axis. In order to combine two binary compounds and make a ternary compound, we desire a minimum variation of lattice spacing. For example, the line connecting AlAs to GaAs is almost vertical. As one moves from AlAs toward GaAs, one would replace a portion of aluminum atoms with Ga atoms. Therefore, at a general point between the two compounds we have $Al_xGa_{1-x}As$. Clearly, $x=1$ at the AlAs point and $x=0$ at the GaAs point. This chart helps us visualize what fraction of certain materials is needed for achieving a certain $E_g$. 
Quantum Well LEDs

On page 121 we saw that the emitted light by LEDs could have a spectrum with line widths that would be too large for certain applications requiring a sharper spectrum. One way to achieve this goal would be to cool the LED. However, for applications where cooling the LED is not an option, a special diode is envisioned which has an extremely thin, narrow bandgap region sandwiched between two wide bandgap materials.

\[ \text{GaAs (narrow } E_g) \quad \text{Al}_x\text{Ga}_{1-x} \text{As} \]

A Quantum-Well LED whose Thickness

\[ d \text{ is very small } \approx 50 \text{ nm} \]

When the thickness of the middle section is about 50 nm, the LED is called a Quantum-Well LED.

Note: At the junction between N and P, and between P and P, any significant concentration of defects would be detrimental to the operation of the LED. To minimize defects at the junction, the lattice spacing of the N-type and P-type material must be very close to the lattice spacing of the middle section P-type material. This goal is achieved by following the chart given on page 123 for engineering the necessary band gap. Any traps at the junction act as a recombination center for the electrons that are injected from the N-region to the P-region.
The discrete energy levels in the conduction and valence bands of semiconductors resemble the energy levels of a hydrogen atom when the material is very very thin. The energy levels become more distinct:

Recall for H-atom: \( E_n = \frac{-E_0}{n^2} \) (Page 2 notes).

Note that for H-atom, the zero energy level was taken to be the highest.

However, in this case, the zero of energy is at the bottom of the conduction band.

This is the case of a two-dimensional electron gas.

Electrons in this thin region form a two-dimensional gas confined between two thick layers.
Apply forward bias to $\text{Al}_x \text{Ga}_{1-x} \text{As}/\text{GaAs}/\text{Al}_x \text{Ga}_{1-x} \text{As}$ heterojunction Quantum Well LED:

Electrons will pour into the QW from the N-side and holes will move from the P-side into the QW as shown in the diagram.

Electrons at the bottom of the well will recombine with the holes at the top of $E_{v1}$ well. As a result a photon of energy $\nu F$ is released.

Now increase the forward bias: more electrons will occupy the lower energy levels of the Quantum Well (QW).

Transitions between $E_1, E_2, E_3, ...$ and $E'_1, E'_2, E'_3, ...$ energy levels must follow quantum mechanical selection rules. Further increase the forward bias: now the QW is flooded with electrons. This will result in transitions from states higher in energy than $E_1$. Therefore the emitted light will involve photons of higher energies, which means the LED emits a beam of broad spectrum. The Quantum Well structure was introduced primarily to create a light output of narrow line width $\Delta \lambda$. Now by increasing the forward bias the sharpness of the emitted light is diminished.

Thus, a single QW LED produces a sharp output line only at low forward bias. For sharp signal outputs, one must modify the above structure.
**Multiple Quantum Well LEDs (MQW)**

To remedy the problem of filling up the single QW under larger forward bias, we can introduce multiple quantum wells.

Due to the existence of multiple quantum wells, the wells do not fill up and most of electrons and holes will reside at energy levels $E_1$ and $E'_1$, respectively. An example of a multiple QW is:

\[
\text{GaN} / \text{In}_x\text{Ga}_{1-x} \text{N} / \text{GaN} / \text{In}_x\text{Ga}_{1-x} \text{N} / \text{GaN} \ldots
\]

$E_{g2} = 3.4 \text{ eV}$

$E_{g1} = 2.7 \text{ eV}$

These are the active layers of the multiple QW LED.

\[
\lambda = \frac{1240}{2.7} \approx 459 \text{ nm} \rightarrow \text{blue}
\]

Comparison of light output power of Single Quantum Wells (SQW) and Multiple Quantum Wells (MQW)
LED "Brightness"

In this section we will examine the human eye's perception of the brightness of light sources. There are two factors:

1. Energy emitted per second (power), and intensity: Power per unit area.
2. The degree to which the human eye senses various wavelengths.

The second factor is based on the responsiveness of the human eye to the spectrum of sunlight at the Earth's surface. This responsiveness is defined by a Gaussian-like luminosity function $V(\lambda)$ which peaks at $\lambda = 555$ nm. $V(\lambda)$ is also called the visibility function.

Clearly, both factors (the light intensity and $V(\lambda)$) are important in our perception of brightness of light sources.

\[
\begin{align*}
\text{Luminous efficiency} & \rightarrow V(\lambda) \\
\text{of the eye} & \\
\end{align*}
\]

1. Emittted radiation power: $P_0$
2. Luminous efficiency of the eye: $V(\lambda)$

Human eye's visual brightness is called the luminous flux $\Phi_f$ and is measured in Lumens (lm) - dimensionless

\[
\Phi_f = P_0 \left( \frac{\text{Watts}}{\text{Watts}} \right) \times 683 \left( \frac{\text{lm}}{\text{Watts}} \right) \times V(\lambda)
\]

$\Phi_f$: scaling factor
In order to determine the visual brightness of the source, we need to know the wavelength of the emitted light so we can determine the factor \( V(\lambda) \) from the graph on the previous page. We also need to know the emission power of the source \( (P_0) \).

**Question:** Suppose an LED emits green light of \( \lambda=555 \text{ nm} \). What power, \( P_0 \), corresponds to 1 lm of brightness?

\[
\Phi_f = 1 \text{ lm}, \quad V(\lambda=555) = 1 \quad \text{according to the graph.}
\]

\[
\Phi_f = 683 \quad P_0 \quad V(\lambda) \Rightarrow 1 = 683 \quad P_0 \quad (1)
\]

\[
\Rightarrow P_0 = \frac{1}{683} \quad \text{Watts} = 1.46 \text{ mW}
\]

So a green light source with the emitting power of 1.46 mW will yield a 1 lm visibility for the human eye.

Commercial lights are categorized based on two factors:

1. # lumens \( \rightarrow \) For example, 900 lm
2. Temperature \( \rightarrow \) For example, 2700 K

The temperature indicates whether the light bulb emits “soft” light similar to the incandescent light bulbs, or “white” light similar to the Sunlight. The higher the temperature, the “whiter” the light.

Next, we will define the Luminous Efficiency of the light source (LED or other light bulbs):
Luminous Efficiency $\eta_{LE}$

Suppose a light bulb has a certain luminosity $\Phi_L$. But, how do we determine if this is an efficient light source? We need to know how much electric power it consumes:

$$\eta_{LE} = \frac{\Phi_L}{IV}$$

**Example 1**: A 60 W incandescent light bulb, typically has a 900 lm rating.

$$\eta_{LE} = \frac{900 \text{ lm}}{60 \text{ W}} \Rightarrow \eta_{LE} = 15 \text{ lm/Watts}$$

**Example 2**: A 100 W incandescent light bulb has a 1700 lm brightness.

$$\eta_{LE} = \frac{1700 \text{ lm}}{100 \text{ W}} = 17 \text{ lm/Watts}$$

**Example 3**: A 10 W LED light bulb has a brightness of 600 lm. What is its luminous efficiency?

$$\eta_{LE} = \frac{600 \text{ lm}}{10 \text{ W}} = 60$$

**Example 4**: Compare a red LED of $\lambda = 650 \text{ nm}$ with optical power of 10 mW with a green LED of $\lambda = 532 \text{ nm}$ with optical power of 5 mW. Which one appears brighter?

Using $V(\lambda)$ curve:

For the red LED: $V(\lambda) \approx 0.1$

$$\Rightarrow \Phi_1 = 683 P_0 V(\lambda_1) = (683)(10 \times 10^{-3})(0.1) = 0.68 \text{ lm}$$

For the green LED: $V(\lambda) \approx 0.83 \Rightarrow \Phi_2 = (683)(5 \times 10^{-3})(0.83) = 2.97 \text{ lm}$

Despite having half the power, the green LED appears more than four times brighter than the red LED.
So far we learned about lumens as the units of luminous flux, which is a measure of perceived brightness of a light source for the human eye.

But, what is Lux?

Lux refers to the intensity of the incident light.

Recall: \[ \text{Intensity} = \frac{\text{Power}}{\text{Area}} \]

Suppose a light bulb has a rating of 900 lumens. This source of light would have a large intensity if all of the emitted power is incident on a small area.

**Example 1**  Assume the entire light output of this light bulb is incident on an area of 1.0 m\(^2\).

\[ \Rightarrow \text{Lux} = \frac{900 \text{ lm}}{1 \text{ m}^2} = 900 \text{ lm/m}^2 \]

**Example 2**  Assume the entire light output is incident on an area of 1 cm\(^2\).

\[ \Rightarrow \text{Lux} = \frac{900}{1 \times 10^{-4}} = 9 \times 10^6 \text{ lm/m}^2 \]

The incident light intensity plays an important role in the design and specifications of light detectors, where the active area of the detector reacts to the number of Lux of light that it receives.
Types of LEDs for Optical Fiber Communication

1. Surface-Emitting LED (SLED)
   - Double heterostructure

2. Edge-Emitting LED (ELED)
   - Double heterostructure

Coupling the LED into:

a. SLED
   - Microlens
   - Multimode fiber
   - Epoxy
   - Active layers
   - Insulator
   - Electrode

b. ELED
   - Epoxy
   - Multimode fiber
   - Insulator
   - Electrode
   - Multimode fiber
   - Single mode fiber

The use of GRIN lens makes it possible to launch the ELED into a single mode fiber.

GRIN → Graded Index
The index of refraction gradually changes from the surface to the axis of the lens.