Handout 1
EE442; Spring 2017

The Definition of 'dB' and 'dBm'

A decibel (dB) in electrical engineering is defined as 10 times the base-10 logarithm of a ratio between two power levels; *e.g.*, P_{out}/P_{in} (gain, in other words). To wit:

$$N\ dB = 10 * \log_{10} (P1/P2)$$

All gains greater than 1 are therefore expressed as positive decibels (>0), and gains of less than 1 are expressed as negative decibels (<0). Note that for cases most of us encounter, the linear ratio of P1/P2 must be a positive number (>0) since the logarithm of 0 is undefined and the logarithm of negatives numbers are complex (they contain both a real and an imaginary part for which we have no meaning for power quantities). The dB value, though, can theoretically take on any value between −∞ and +∞, including 0, which is a gain of 1 [10 * log (1) = 0 dB].

'dBm' is a decibel-based unit of power that is referenced to 1 mW. Since 0 dB of gain is equal to a gain of 1, 1 mW of power is 0 dB greater than 1 mW, or 0 dBm. Similarly, a power unit of dBW is decibels relative to 1 W of power.

1 mW = 0 dBm

Accordingly, all dBm values greater than 0 are larger than 1 mW, and all dBm values less than 0 are smaller than 1 mW (see Fig. 1). For instance, +3.01 dBm is 3.01 dB greater than 1 mW; *i.e.*, or 0 dBm + 3.01 dB = +3.01 dBm (2 mW). −3.01 dBm is 3.01 dB less than 1 mW; *i.e.*, or 0 dBm + (−3.01) dB = −3.01 dBm (0.5 mW).

The following table gives some numerical examples so you can see the correlation between mW and dBm. The same set of values plotted on a logarithmic scale would produce a straight line. Because of the logarithmic relationship, the graph bunches the smaller values against the left vertical axis. A magnified version of the 0 to 1 mW region is inset for clarity.

	Power	
	mW	dBm
1μW →	0.001	-30
	0.002	-27
	0.005	-23
10μW →	0.01	-20
	0.02	-17
	0.05	-13
100μW →	0.1	-10
	0.2	-7
	0.5	-3
1mW →	1	0
	2	3
	5	7
10mW →	10	10
	15	12
	20	13
	50	17
100mW →	100	20
	200	23
	500	27
1W →	1000	30

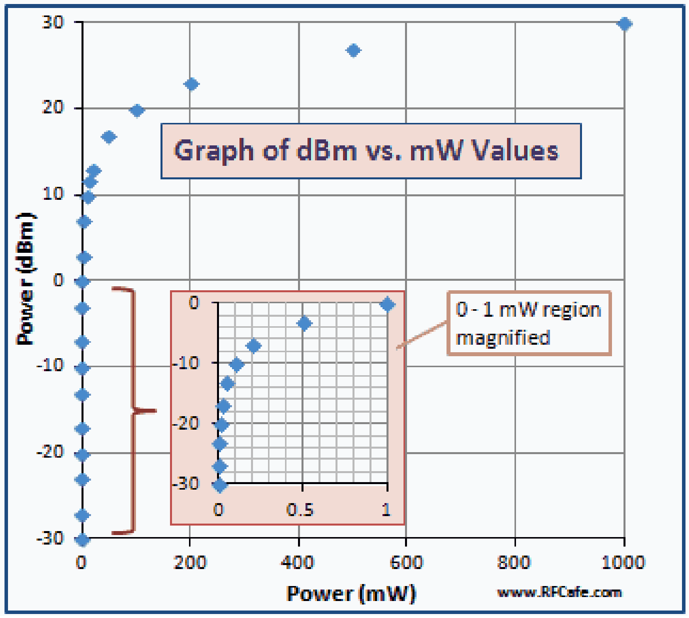

Fig. 1 - Graph of Power in Units of dBm vs. mW

Fig. 2 is a table and graph of dB vs. linear gain ratios similar to the dBm vs. mW in Fig. 1. Note that the numbers and curves are exactly the same; only the axis labels are changed. That is because dBm is a unit of power expressed in dB relative to 1 mW (0 dBm).

Output/Input Gain		
Linear		dB
1/1000	0.001	-30
1/500	0.002	-27
1/200	0.005	-23
1/100	0.01	-20
1/50	0.02	-17
1/20	0.05	-13
1/10	0.1	-10
1/5	0.2	-7
1/2	0.5	-3
1	1	0
2	2	3
5	5	7
10	10	10
15	15	12
20	20	13
50	50	17
100	100	20
200	200	23
500	500	27
1000	1000	30

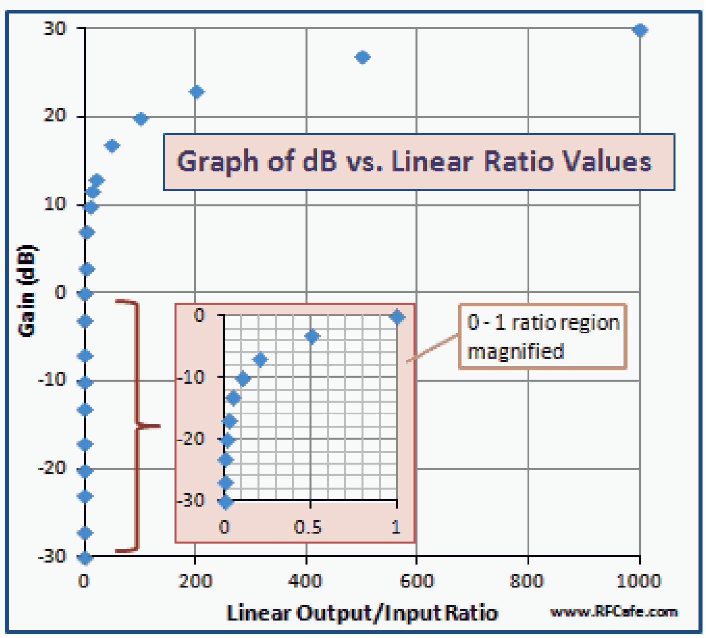

Fig. 2 - Graph of Gain in Units of dBm vs. Linear Ratio

Linear Gain (output/input ratio) vs. Logarithmic (decibels, dB) Gain

Fundamentally, gain is a multiplication (or division) factor. As an example, an amplifier might have a gain that increases the signal by a factor of 4 (*i.e.*, 4x) from input to output (see Fig. 3). If a 1 mW (0 dBm) signal is fed into the amplifier, then 1 mW * 4 = 4 mW comes out. In terms of decibels, a factor of 4 is equivalent to 10 * log (4) = 6.02 dB, so 0 dBm in plus 6.02 dB of gain yields +6.02 dBm at the output.

1 mW * 4 = 4 mW

0 dBm + 6.02 dB = 6.02 dBm

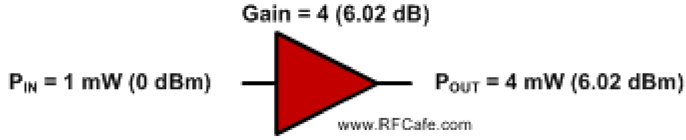

Fig. 3 - Single amplifier gain.

Combining Gains (linear and dB) w/Positive Values

If an amplifier with a gain of 4 is in series with a second amplifier with a gain of 6, then the total gain is 4 * 6 = 24. In terms of decibels, a factor of 6 is equivalent to 10 * log (6) = 7.78 dB, and a factor of 24 is equivalent to 10 * log (24) = 13.8 dB.

Just as 4 x 6 = 24 (linear gain), 6.02 dB + 7.78 dB = 13.8 dB (decibel gain).

If a 1 mW signal (0 dBm) is fed into the amplifier, then 4 mW comes out of the first amplifier, and 24 mW comes out of the second amplifier. See Fig. 4.

1 mW * 4 * 6 = 24 mW

0 dBm + 6.02 dB + 7.78 dB = 13.8 dBm

Fig. 4 - Cascaded dual amplifier gain.

Combining Gain and Loss (linear and dB)

This next example shows what happens when a gain < 1 (a loss) is encountered, where an attenuator with a gain of 1/6 is placed after the first amplifier instead of having a second amplifier. See Fig. 5.

Thus, 4 * 1/6 = 2/3 (linear gain). Similarly 6.02 dB - 7.78 dB = −1.76 dB (decibel gain). As with the previous example, if a 1 mW signal (0 dBm) is fed into the amplifier with a gain of 4, then 4 mW comes out. That 4 mW then goes into the attenuator with a linear gain of 1/6 and comes out at a power level of 4/6 mW (2/3 mW). The total gain in this case is 4/6 = 2/3, so the output power will actually be less than the input power.

1 mW * 4 * 1/6 = 2/3 mW = 0.67 mW

0 dBm + 6.02 dB + (-7.78 dB) = −1.76 dBm

Fig. 5 - Cascaded amplifier gain and attenuator.

Note that power levels greater than 0 dBm sometimes include the 'plus' sign (+) in order to emphasize that it is not negative. This is particularly so when power levels are displayed on a block diagram where both positive and negative values are present.

Summary

When making power measurements in the laboratory or in the field, most people find it easier to add and subtract gains and power levels than to multiply and divide gains and power levels. dB and dBm units make that possible. The important thing to remember is to ***never mix*** linear gain (ratio) units and wattage power (mW) units with logarithmic gain (dB) and power (dBm) units.

Quantities must be either in all linear or all decibel units. The following type of calculation is ***NOT*** allowed because it mixes linear values with logarithmic values.

12 mW + 34 mW + 8 mW + 20 dB

Reference: http://www.rfcafe.com/references/electrical/decibel-tutorial.htm