Consider the following demand and cost functions for a monopolistic firm. The industry demand is given by \( Q = 1000 - 5P \). The firm’s total cost is given by \( TC(q) = 300 + \frac{q^2}{3} \). Assume this represents both the short run and long run costs of the firm.

(a) Find the equilibrium price and quantity for a single price monopolist.

To solve for the monopolistic outcome you need to find the marginal revenue function. \( Q^D = 1000 - 5P \) which, solving for price gives \( P = 200 - \frac{1}{5}Q \).

\( TR = P \times Q \), so \( TR = 200Q - \frac{1}{5}Q^2 \).

Marginal revenue is thus, \( MR = \frac{\partial TR}{\partial Q} = 200 - 2/5Q \).

Setting \( MR = MC \),

\( 200 - 2/5Q = 2/3Q \), and solving for \( Q \) gives the monopolist output of \( Q^* = 187.5 \).

The highest price consumers are willing to pay is \( P = 200 - \frac{1}{5}(187.5) = 162.50 \). \( P^* = $162.50 \).

(b) Calculate the average total cost at the profit maximizing output.

\( ATC(187.5) = $64.10 \)

(c) Calculate the amount of the monopoly profits per unit.

\( \pi/Q = 162.50 - 64.10 = $98.40 \) per unit profit.

(d) Calculate the amount of the monopoly total profits.

Total profit \( \pi = 98.40(187.5) = $18,450 \).

(e) Find the competitive equilibrium.

The competitive equilibrium is where supply equals demand. Since \( MC = S \), where \( MC = 2Q/3 \)

\( Q^D = 1000 - 5P \) which, solving for price gives \( P = 200 - 1/5Q \).

Set \( S = D \)

\( 200 - 1/5Q = 2Q/3 \).

\( Q = 230.77 \)

\( P = 153.85 \)

(f) Calculate the amount of consumer surplus under the single price monopoly.

Under Monopoly, \( C.S. = \frac{1}{2}(187.5)(200-162.5) = $3,515.63 \).

(g) Calculate the amount of consumer surplus under perfect competition. Compare consumer surplus under monopoly and perfect competition. Which is greater?

Under Perfect Competition \( C.S. = \frac{1}{2}(230.8)(200-153.84) = $5,326.86 \)

(h) Calculate the amount of producer surplus under the single price monopoly.

Under Monopoly, \( P.S. = (162.5-125)187.5 = 7031.25 \)

\[ + \frac{1}{2}(187.50)(125) = 3906.25 \]

\( P.S. = $18,750 \)
(i) Calculate the amount of producer surplus under perfect competition. Compare producer surplus under monopoly and perfect competition. Which is greater?

Under Perfect Competition, \( P.S. = \frac{1}{2}(230.8)(153.84) = \$17,753.14. \)

(j) Calculate the amount of the deadweight loss. What causes the deadweight loss?

\[ \text{DWL} = \frac{1}{2}(230.8 - 187.5)(162.2 - 125) = \$811.88. \]

The dead weight loss results from the monopolist reducing output below the competitive level.

(k) Show the above answers graphically.

(2) Suppose now that the monopolist engages in perfect (first degree) price discrimination.

(a) What is the equilibrium price and quantity of the perfectly price discriminating monopolist.

There is no single price since every consumer is charged their maximum willingness to pay, while the equilibrium output will be the same as the competitive output, \( Q = 230.8. \)

(b) What is the amount of consumer surplus. Compare the consumer surplus with that calculated under perfect competition and the single price monopoly.

There is no consumer surplus. All the consumer surplus is extracted and retained by the monopolist. Consumer surplus is greater under perfect competition and a single price monopolist.

(c) What is the amount of producer surplus. Compare the consumer surplus with that calculated under perfect competition and the single price monopoly.

\[
\begin{align*}
P.S. &= \frac{1}{2}(230.8)(200-153.84) = \$5,326.86 \\
&+ \frac{1}{2}(230.8)(153.84) = \$17,753.14. \\
P.S. &= \$23,080.
\end{align*}
\]
(d) Calculate the amount of the deadweight loss. Compare the dead weight loss with that calculated under perfect competition and single price monopoly.

There is no dead weight loss since the competitive output is produced.

(e) Show the above answers graphically.

(3) Suppose instead that the monopolist wants to engage in second degree price discrimination by employing a two part tariff.

Suppose also that the market consists of two types of consumers that can be represented by the following demand equations.

Consumer Type One: \( Q_1 = 500 - P_1 \)

Consumer Type Two: \( Q_2 = 500 - 4P_2 \)

Assume that output can be produced at a constant marginal cost of $10 which also equals average total cost.

(a) If the monopolist wants to maintain both types of consumers, find the price per unit and the amount of the tariff that will maximize the monopolists total profits.

With heterogeneous consumers, the profit maximizing tariff and per unit price is found by trial and error. Starting with a $10 price per unit the maximum entry fee that can be charged while still retaining both consumers is equal to the consumer surplus of the marginal consumer, consumer two, which is C.S. = \( \frac{1}{2} (460)(125-10) = 26,450 \).

Total profit \( \pi = 2(26,450) = 52,900 \), which consist solely of the tariff since the price per unit is set at the zero profit price.

Trying different price per unit and tariff combinations produces the profit maximizing price per unit of P = $25 and a tariff of $20,000.
(b) Show the above answer graphically. Shown below.

(c) If the monopolist wants to retain only its most profitable consumers, find the price per unit and the amount of the tariff that will maximize the monopolist's total profits.

Consumer One has the most inelastic demand curve and is thus the most profitable. The profit maximizing price is set at the competitive price per unit $P = $10 and the profit maximizing tariff is $120,050 which is equal to the total profit.

(d) Show the above answer graphically. Shown above.

(e) Which pricing strategy would you suggest, (a) or (c)? Explain your answer.

In this case, concentrating only on your most profitable consumers maximizes profits,

(4) Suppose that the market consist of two main groups who differ based on their price elasticities of demand. The demand functions of each group are as follows:

(a) What market conditions must exist in order to engage in third degree price discrimination?

(b) Derive the marginal revenue function for market one.

$Q_1 = 500 - P_1$ solve the demand curve for $P$, $P = 500-Q_1$.

$TR = 500Q - Q^2$  

$MR= 500 - 2Q$
(c) Calculate the profit maximizing price, output and profits in market one.
Set MR=MC
500-2Q = 10
Q = 245.
P = 500-245
P = $255.
π/Q= 255-10=245
π = 245(245) = $60,025

(d) Derive the marginal revenue function for market two.
Q₂ = 500 - 4P₁, solve the demand curve for P, P = 125-(1/4)Q₁.
TR = 125Q - (1/4)Q₁²
MR = 125 - ½Q

(e) Calculate the profit maximizing price, output and profits in market two.
Set MR=MC
125 - ½Q = 10
Q = 230.
P = 125-1/4(230)
P = $67.50.
π/Q = 67.50 -10 = $57.50
π = 57.50(230) = $13,225

(f) Show your answers for a-e above graphically. Shown above.