Suppose you have three individuals in a community with the following demands for fire protection in hours per month:
\[ q_1 = 300 - \frac{P}{5} \]
\[ q_2 = 300 - \frac{P}{7} \]
\[ q_3 = 420 - \frac{P}{5} \]

The marginal cost of fire protection is given by \( MC = 2500 + 2Q \)

(a) Derive the competitive equilibrium provision of fire protection per month. Explain fully and show graphically.

The Nash-Cournot competitive equilibrium is found by horizontally summing the demand curves.

\[ Q^D = \sum_{i=1}^{3} q_i = 1020 - \frac{19}{35}P \]

To solve for the equilibrium quantity solve for price and set equal to \( MC \)

\[ \frac{35700}{19} - \frac{35}{19}Q = 2500 + 2Q \]

which gives us: \( Q = \frac{11800}{73} \)

Since output cannot be negative \( Q = 0 \), the public good is not provided in a competitive market. This can be seen by graphing the functions.
(b) Derive the efficient provision of fire protection per month using the Samuelson rule. Explain fully and show graphically.

The efficient provision is found by the Samuelson condition $\sum MB = MC$, where the sum of the marginal benefits is found by vertically summing the demands at each quantity.

$$\sum MB = 5700 - 17Q,$$

setting this equal to marginal cost gives,

$$5700 - 17Q = 2500 + 2Q$$

$$Q^E = 168.42$$

Shown graphically

(c) Why do the levels of fire protection differ in questions a & b? Explain fully.
Suppose that it is decided to finance the efficient level of fire protection through voluntary contributions.

(d) If a “head” tax is used, what is the amount of voluntary contribution needed to finance the efficient level of fire protection?

With a head tax each pays an equal proportion of the cost. At the equilibrium provision of \( Q = 168.42 \), the marginal cost of production is $2,836.86. Divided equally among the three, each would pay \( 2836.83/3 = $945.62 \) per unit.

(e) Will the efficient level of fire protection be provided if each member of the community is asked to contribute the amount in part d? Explain why or why not.

To see if they will voluntarily contribute to the public good you have to compare the marginal benefit each gets from consumption with the marginal cost which is the head tax. The marginal benefit of consumption at the efficient provision is

\[
MB_1 = 1500-5(168.42) = $657.90 \text{ for consumer 1.}
\]
\[
MB_2 = 2100-7(168.42) = $921.06 \text{ for consumer 2.}
\]
\[
MB_3 = 2100-5(168.42) = $1257.90 \text{ for consumer 3.}
\]

Only consumer 3 will voluntarily pay the head tax of $945.62 toward the provision of the public good, so the public good does not get provided.

(f) If a “Lindahl” tax is used, what is the amount of voluntary contribution needed to finance the efficient level of fire protection?

A Lindahl tax scheme charges each consumer a tax equal to the marginal benefit of consumption. From above you can see that,

Consumer one should be charged \( MB_1 = 1500-5(168.42) = $657.90 \) for each unit.
Consumer two should be charged \( MB_2 = 2100-7(168.42) = $921.06 \) for each unit.
Consumer three should be charged \( MB_3 = 2100-5(168.42) = $1257.90 \) for each unit.

(g) Does the Lindahl tax result in a shortage or surplus of funds needed to cover the cost of the efficient level of fire protection?

Consumer one contributes \( $657.90 \times 168.42 = $110,803.52 \) towards the public good.
Consumer two contributes \( $921.06 \times 168.42 = $155,124.93 \) towards the public good.
Consumer three contributes \( $1257.90 \times 168.42 = $211,855.52 \) towards the public good.
Total contributions equal = $477,783.97.

Total cost of providing the efficient level of public good is given by the area under the marginal cost curve up to the quantity provided.

This can be found by calculating the area of the triangle \( \frac{1}{2}(168.42)(2836.86-2500) = $28,366.98 \) and the area of the rectangle \( (2500)(168.42) = $421,050 \).
\[ TC = 449,416.98. \]

There is a surplus of \( 477,783.96 - 449,416.98 = $28,366.98 \), which can be redistributed back to the consumers.