7. PLANE WAVE PROPAGATION
Review

- We have learned about wave propagation
  - Guided propagation
    - Skywave \([f=3-3\text{- MHz}]\)
    - Cable
  - Transmission Line
    - Reflected wave
      - Constructive Parameters
      - Standing waves
    - Wave equation
      - Time representation
      - Phasor representation
      - Propagation constant

We have been assuming TEM waves:
- Direction of propagation is in \(Z\)
- \(E\) is in \(r\) direction
- \(H\) is radial
Sky Wave (Skip) Propagation

Rays entering at more than critical angle pass through ionosphere.

First refraction

Second refraction

Ionospheric layer

Station 1

Station 2

Critical angle

First reflection

Skip zone — no signal can be received here

Ground and space wave region — signals received here
EM Waves can be unguided:
1. EM Source radiates → Spherical wave
2. Spherical wave → Planer wave (far-field effect)
3. Planer waves are **uniform**
4. We consider **TEM waves**
Planer Waves

Unbounded EM Waves:
1. Waves are traveling in dielectric (perfect dielectric → lossless media)
2. We use wave equations instead of transmission line equations
3. We refer to intrinsic impedance rather than characteristic impedance, Zo
4. Propagation constant = loss + Phase constant
5. \( k = \text{wave number} \) (same as phase constant in transmission line)

\[
p = \frac{\omega}{k} = \frac{\omega}{\omega \sqrt{\mu \varepsilon}} = \frac{1}{\sqrt{\mu \varepsilon}} \quad \text{(m/s)},
\]

\[
\lambda = \frac{2\pi}{k} = \frac{u_p}{f} \quad \text{(m)}.
\]

We start by considering phasor form!
Review of Maxwell’s Equations

We now express these in phasor form. HOW?

<table>
<thead>
<tr>
<th>POINT FORM</th>
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<tbody>
<tr>
<td>$\nabla \times H = J_c + \frac{\partial D}{\partial t}$</td>
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THIS IS WHAT WE HAVE LEARNED SO FAR......

We now express these in phasor form. HOW?
Review of Maxwell’s Equations –
General Form

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\[
\begin{align*}
\nabla \cdot \tilde{\mathbf{E}} &= \tilde{\rho}_v / \varepsilon, \\
\nabla \times \tilde{\mathbf{E}} &= -j \omega \mu \tilde{\mathbf{H}}, \\
\nabla \cdot \tilde{\mathbf{H}} &= 0, \\
\nabla \times \tilde{\mathbf{H}} &= \tilde{\mathbf{J}} + j \omega \varepsilon \tilde{\mathbf{E}}.
\end{align*}
\]

- All the fields are in phasor form
- Time derivatives are expressed differently: $d/dt \rightarrow j \omega$
Maxwell’s Equations – Free Space Set

- We assume there are **no charges** in free space and thus, \( J_c = \sigma E = 0 \)

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<td>( \nabla \times H = \frac{\partial D}{\partial t} )</td>
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*Time-varying E and H cannot exist independently!* If \( \frac{dE}{dt} \) non-zero \( \Rightarrow \frac{dD}{dt} \) is non-zero \( \Rightarrow \) Curl of H is non-zero \( \Rightarrow \) H is non-zero

*If H is a function of time \( \Rightarrow \) E must exist!*
Maxwell’s Equations – Free Space Set

- We assume there are **no charges** in free space and thus, $j_c = \sigma E = 0$

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We will use these to derive the wave equation for EM waves.
Wave Equations -

Assume no volume charges

\[ \nabla \cdot \vec{E} = 0, \]
\[ \nabla \times \vec{E} = -j \omega \mu \vec{H}, \]
\[ \nabla \cdot \vec{H} = 0, \]
\[ \nabla \times \vec{H} = j \omega \varepsilon_c \vec{E}. \]

\[ \nabla \times (\nabla \times \vec{E}) = -j \omega \mu (\nabla \times \vec{H}). \]
\[ \nabla \times (\nabla \times \vec{E}) = -j \omega \mu (j \omega \varepsilon_c \vec{E}) = \omega^2 \mu \varepsilon_c \vec{E}. \]

**Special Property:**
\[ \nabla \times (\nabla \times \vec{E}) = \nabla (\nabla \cdot \vec{E}) - \nabla^2 \vec{E}, \]
\[ \nabla^2 \vec{E} = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \vec{E}. \]

\[ \nabla^2 \vec{E} + \omega^2 \mu \varepsilon_c \vec{E} = 0, \]

Complex permittivity
\[ \varepsilon_c = \varepsilon - j \frac{\sigma}{\omega}, \]
Homogeneous Wave Equations for E and H

\[ \nabla^2 \mathbf{E} + \omega^2 \mu \varepsilon_c \mathbf{E} = 0 \]

Propagation Constant:

\[ \gamma^2 = -\omega^2 \mu \varepsilon_c, \]

Complex permittivity

\[ \varepsilon_c = \varepsilon - j \frac{\sigma}{\omega}, \]

Similarly:

\[ \nabla^2 \mathbf{H} - \gamma^2 \mathbf{H} = 0. \]
Homogeneous Wave Equations for E and H (Lossless case)

\[ \nabla^2 \tilde{E} + \omega^2 \mu \varepsilon_c \tilde{E} = 0 \]

Propagation Constant:

\[ \gamma^2 = -\omega^2 \mu \varepsilon_c, \]

Complex permittivity

\[ \varepsilon_c = \varepsilon - j \frac{\sigma}{\omega}, \]

Note: if lossless conductivity = 0

\[ \gamma^2 = -\omega^2 \mu \varepsilon. \]

\[ k = \omega \sqrt{\mu \varepsilon}. \]

\[ \nabla^2 \tilde{E} + k^2 \tilde{E} = 0. \]
Our assumption was having a uniform plane wave.

A uniform plane wave is characterized by electric and magnetic fields that have uniform properties at all points across an infinite plane.

- There is no change of field.
  - For example, in x-y plane: \( \frac{dE}{dx} = \frac{dE}{dy} = 0 \)
Uniform Plane Wave (x-y plane)

Consider vector field $\mathbf{E}$:

$$
\mathbf{E} = \hat{x}\mathbf{E}_x + \hat{y}\mathbf{E}_y + \hat{z}\mathbf{E}_z,
$$

$$
\nabla^2 \mathbf{E} + k^2 \mathbf{E} = 0.
$$

$$
\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \left( \mathbf{E}_x \right) + k^2 \left( \mathbf{E}_x \right) = 0.
$$

Must satisfy $\rightarrow$

There is no change in $X$ and $Y$ (uniform)

Same thing for $\mathbf{E}_y$ and $\mathbf{E}_z$:  

Only non-zero vector component

Same thing for $\mathbf{E}_y$ and $\mathbf{E}_z$: 

$$
\frac{d^2 \mathbf{E}_x}{dz^2} + k^2 \mathbf{E}_x = 0.
$$
Uniform Plane Wave (x-y plane) - Solution

General Form of the Solution:
\[ \frac{d^2 \tilde{E}_x}{dz^2} + k^2 \tilde{E}_x = 0. \]

Application of \( \nabla \times \tilde{E} = -j \omega \mu \tilde{H} \) yields:
\[ \tilde{H}_y(z) = \frac{k}{\omega \mu} E^+_{x0} e^{-jkz} = H^+_{y0} e^{-jkz} \]

Summary: This is a plane wave with
\[ \tilde{E}(z) = \hat{x} \tilde{E}^+_x(z) = \hat{x} E^+_{x0} e^{-jkz}, \]
\[ \tilde{H}(z) = \hat{y} \frac{\tilde{E}^+_x(z)}{\eta} = \hat{y} \frac{E^+_{x0}}{\eta} e^{-jkz}. \]
Power and Impedance

Intrinsic Impedance of a lossless medium (analogy to Zo)

\[ \eta = \frac{\omega \mu}{k} = \frac{\omega \mu}{\omega \sqrt{\mu \varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} \quad (\Omega) \]
Time Domain Representation

TEM Traveling Wave Solution

\[ \mathbf{E}(z) = \hat{x}\mathbf{E}_x^+(z) = \hat{x}E_{x0}^+ e^{-jkz}, \]
\[ \mathbf{H}(z) = \hat{y}\frac{\mathbf{E}_x^+(z)}{\eta} = \hat{y}\frac{E_{x0}^+}{\eta} e^{-jkz}. \]

\[ E_{x0}^+ = |E_{x0}^+| e^{j\phi^+} \]

\[ \mathbf{E}(z, t) = \Re \left[ \mathbf{E}(z) e^{j\omega t} \right] \]

Time-Domain Solution

\[ \mathbf{E}(z, t) = \Re \left[ \mathbf{E}(z) e^{j\omega t} \right] \]
\[ = \hat{x}E_{x0}^+ \cos(\omega t - kz + \phi^+) \quad (V/m), \]
and

\[ \mathbf{H}(z, t) = \Re \left[ \mathbf{H}(z) e^{j\omega t} \right] \]
\[ = \hat{y}\frac{|E_{x0}^+|}{\eta} \cos(\omega t - kz + \phi^+) \quad (A/m). \]
Check the Simulator
Directional Relation Between $E$ and $H$

For Any TEM Wave

\[
\begin{align*}
\tilde{H} &= -\frac{1}{\eta} \hat{k} \times \tilde{E}, \\
\tilde{E} &= -\eta \hat{k} \times \tilde{H}.
\end{align*}
\]

Phasor Form

Note:
$E$ and $H$ may have x & y components.
However, they travel in Z direction and
They are perpendicular to each other!
What is $k$? (it is a function of what? Which direction is it pointing at?)
What is $E$?
What is $H$?

K(z) in $+Z$ direction
$E(z,t)$ in $+X$ direction
$H(z,t)$ in $+Y$ direction
Example

The electric field of a 1-MHz plane wave traveling in the +z-direction in air points along the x-direction. If this field reaches a peak value of $1.2\pi$ (mV/m) at $t = 0$ and $z = 50$ m, obtain expressions for $\mathbf{E}(z, t)$ and $\mathbf{H}(z, t)$, and then plot them as a function of $z$ at $t = 0$.

Find $l$, $k$, $E(z,t)$, $H(z,t)$

$$
\lambda = \frac{c}{f} = \frac{3 \times 10^8}{1 \times 10^6} = 300 \text{ m},
$$

$$
k = \left(\frac{2\pi}{300}\right) \text{ (rad/m)}.\n$$

$$
\mathbf{E}(z, t) = \hat{x}|E_{x_0}^+| \cos(\omega t - kz + \phi^+)
$$

$$
= \hat{x} 1.2\pi \cos \left(2\pi \times 10^6 t - \frac{2\pi z}{300} + \phi^+\right) \text{ (mV/m)}.\n$$

$$
-\frac{2\pi \times 50}{300} + \phi^+ = 0 \quad \text{or} \quad \phi^+ = \frac{\pi}{3}\n$$
Example cont.

Hence,

$$\mathbf{E}(z, t) = \hat{x} 1.2\pi \cos \left( 2\pi \times 10^6 t - \frac{2\pi z}{300} + \frac{\pi}{3} \right) \quad (\text{mV/m}),$$

and from Eq. (7.34b) we have

$$\mathbf{H}(z, t) = \hat{y} \frac{E(z, t)}{\eta_0}$$

$$= \hat{y} 10 \cos \left( 2\pi \times 10^6 t - \frac{2\pi z}{300} + \frac{\pi}{3} \right) \quad (\mu\text{A/m}),$$
Check out the Simulator
Polarization - General

- Polarization is the orientation of electric field component of an electromagnetic wave relative to the Earth’s surface.
- Polarization is important to get the maximum performance from the antennas.
- There are different types of polarization (depending on existence and changes of different electric fields):
  - Linear
    - Horizontal (E field changing in parallel with respect to earth’s surface)
    - Vertical (E field going up/down with respect to earth’s surface)
    - Dual polarized
  - Circular (Ex and Ey)
    - Similar to satellite communications
    - TX and RX antennas must agree on direction of rotation
  - Elliptical
- Linear polarization is used in WiFi communications

Polarization can change as the signal travels away from the source!
- Due to the magnetic field of Earth (results in Faraday rotation)
- Due to reflection
Polarization - General

- Polarization is important to get the maximum performance from the antennas
  - The polarization of the antennas at both ends of the path must use the same polarization
  - This is particularly important when the transmitted power is limited
Wave Polarization

The polarization of a uniform plane wave describes the locus traced by the tip of the $\mathbf{E}$ vector (in the plane orthogonal to the direction of propagation) at a given point in space as a function of time.
Wave Polarization

Plane wave propagating along +z:

\[
\mathbf{E}(z) = \mathbf{\hat{x}} E_x(z) + \mathbf{\hat{y}} E_y(z),
\]

\[
\begin{align*}
E_x(z) &= E_{x0} e^{-jkz}, \\
E_y(z) &= E_{y0} e^{-jkz},
\end{align*}
\]

If: \[ E_{x0} = a_x, \]
\[ E_{y0} = a_y e^{i\delta}, \]

then

\[
\mathbf{E}(z) = (\mathbf{\hat{x}} a_x + \mathbf{\hat{y}} a_y e^{i\delta}) e^{-jkz},
\]

\[
\frac{d^2 \tilde{E}_x}{dz^2} + k^2 \tilde{E}_x = 0.
\]

\[
E(z, t) = \Re\left[ \tilde{E}(z) e^{j\omega t} \right]
= \mathbf{\hat{x}} a_x \cos(\omega t - kz)
+ \mathbf{\hat{y}} a_y \cos(\omega t - kz + \delta).
\]
Polarization state describes the trace of $\mathbf{E}$ as a function of time at a fixed $z$.

\[
\widetilde{\mathbf{E}}(z) = (\hat{x}a_x + \hat{y}a_y e^{j\delta})e^{-jkz},
\]

\[
\mathbf{E}(z, t) = \Re \left[ \widetilde{\mathbf{E}}(z) e^{j\omega t} \right] = \hat{x}a_x \cos(\omega t - kz) + \hat{y}a_y \cos(\omega t - kz + \delta).
\]

**Magnitude of $\mathbf{E}$**

\[
|\mathbf{E}(z, t)| = \left[ E_x^2(z, t) + E_y^2(z, t) \right]^{1/2} = \left[ a_x^2 \cos^2(\omega t - kz) + a_y^2 \cos^2(\omega t - kz + \delta) \right]^{1/2}
\]

**Inclination Angle**

\[
\psi(z, t) = \tan^{-1} \left( \frac{E_y(z, t)}{E_x(z, t)} \right)
\]
Linear Polarization:

$\delta = 0 \quad \text{or} \quad \delta = \pi$

In-phase \quad Out-of-phase

\[ E(z, t) = \Re \left[ \tilde{E}(z) e^{j\omega t} \right] \]
\[ = \hat{x}a_x \cos(\omega t - kz) + \hat{y}a_y \cos(\omega t - kz + \delta). \]
\[ = (\hat{x}a_x + \hat{y}a_y) \cos(\omega t - kz) \quad \text{(in-phase)}, \]
\[ = (\hat{x}a_x - \hat{y}a_y) \cos(\omega t - kz) \quad \text{(out-of-phase)}. \]

If $a_y = 0$, then $\psi = 0^\circ$ or $180^\circ$, and the wave is $x$-polarized; conversely, if $a_x = 0$, then $\psi = 90^\circ$ or $-90^\circ$, and the wave is $y$-polarized.
Circular Polarization

LHP: $a_x = a_y = a$ and $\delta = \pi/2$.

RHP: $a_x = a_y = a$ and $\delta = -\pi/2$.
LH Circular Polarization

$$\mathbf{E}(z, t) = \Re \left[ \tilde{\mathbf{E}}(z) e^{j \omega t} \right]$$

$$= \hat{x} a \cos(\omega t - k z) + \hat{y} a \cos(\omega t - k z + \pi/2)$$

$$= \hat{x} a \cos(\omega t - k z) - \hat{y} a \sin(\omega t - k z).$$

**Magnitude**

$$|\mathbf{E}(z, t)| = \left[ E_x^2(z, t) + E_y^2(z, t) \right]^{1/2}$$

$$= [a^2 \cos^2(\omega t - k z) + a^2 \sin^2(\omega t - k z)]^{1/2}$$

$$= a,$$

**Inclination Angle**

$$\psi(z, t) = \tan^{-1} \left[ \frac{E_y(z, t)}{E_x(z, t)} \right]$$

$$= \tan^{-1} \left[ \frac{-a \sin(\omega t - k z)}{a \cos(\omega t - k z)} \right]$$

$$= -(\omega t - k z).$$
RH Circular Polarization:

\[ a_x = a_y = a \] and \[ \delta = -\pi/2 \].

(a) LHC polarization

(b) RHC polarization
Example
**LCD**

Liquid Cristal
Molecular spiral
Operation of a Single Pixel

Bright pixel

Liquid crystal

Molecule of liquid crystal

Polarizing filter

Dark pixel

5 micron
LCD 2-D Array