Chapter 4

Bandpass Signaling
Outline

• Complex Envelope Representation of Bandpass Waveforms,
• Representation of Modulated Signals,
• Spectrum of Bandpass Signals,
• Evaluation of Power,
• Bandpass Filtering and Linear Distortion,
• Bandpass Sampling Theorem,
• Received Signal Plus Noise,
• Classification of Filters and Amplifiers,
• Nonlinear Distortion, Limiters, Mixers, Up Converters, and Down Converters,
• Frequency Multipliers, Detector Circuits, Phase-Locked Loops and Frequency Synthesizers, Transmitters and Receivers,
• Software Radios
Baseband & Bandpass Waveforms

- **A baseband waveform** has a spectral magnitude that is nonzero for freq in the vicinity of the origin (f=0) and negligible elsewhere.
  - It is a signal whose range of freq is measured from zero to a maximum bandwidth
  - E.g., an audio signal from a microphone, a TTL signal from a digital circuit.

- **A bandpass waveform** has a spectral magnitude that is nonzero for freq in some band concentrated about a freq $f = \pm f_c$.
  - The spectral magnitude is negligible elsewhere.
  - $f_c$ is called carrier freq.
  - E.g., An AM radio signal that broadcast news over $f_c=850$ kHz is a bandpass signal
Modulation is the process of imparting the source information onto a bandpass signal with carrier freq $f_c$ using amplitude or phase perturbation (or both).

- The bandpass signal is called modulated signal $s(t)$.
- The baseband signal is called modulating signal $m(t)$.

Bandpass communication signal is obtained by modulating a baseband analog or digital signal on a carrier.

- Whereas baseband signal cannot go far, a bandpass signal goes a long distance.
Modulating & Modulated Signals
Complex Envelope Representation

- Let \( v(t) \) represent bandpass waveforms
  - \( v(t) \) can represent the signal when \( s(t) = v(t) \), noise when \( n(t) = v(t) \), filtered signal + noise when \( r(t) = v(t) \).
- A physical bandpass waveform can be represented by \( v(t) = \Re\left\{ g(t)e^{j\omega_c t}\right\} \)
  - where \( g(t) \) is called the complex envelope of \( v(t) \), \( \omega_c = 2\pi f_c \).

\[
g(t) = x(t) + jy(t) = |g(t)|e^{j\angle g(t)} = R(t)e^{j\theta(t)}
\]

- \( e^{j\omega_c t} \) factor shifts (translates) the spectrum of the baseband \( g(t) \) signal from baseband up to carrier freq \( f_c \).
- \( R(t) \) is said to be amplitude modulation (AM) on \( v(t) \).
- \( \theta(t) \) is said to be phase modulation (PM) on \( v(t) \).
Representation of Modulated Signal

- Modulation is the process of encoding the source information $m(t)$ into a bandpass signal $s(t)$.

- The modulated signal is an application of bandpass representation, i.e., $s(t) = \text{Re}\{g(t)e^{j\omega_c t}\}$

- The complex envelope $g(t)$ is a function of the modulating signal $m(t)$, i.e., $g(t) = g[m(t)]$
  - E.g., for AM modulation, $g[m(t)] = A_d[1 + m(t)]$
Spectrum of Bandpass Signal

**Theorem:** If the bandpass waveform is represented by \( v(t) = \text{Re}\{g(t)e^{j\omega_c t}\} \)

then the spectrum of the bandpass waveform is

\[
V(f) = \frac{1}{2} [G(f - f_c) + G^*(-f - f_c)] \quad \& \quad \text{PSD} = P_v(f) = \frac{1}{4} [P_g(f - f_c) + P_g(-f - f_c)]
\]

**Proof for \( V(f) \):**

\[
v(t) = \text{Re}\{g(t)e^{j\omega_c t}\} = \frac{1}{2}g(t)e^{j\omega_c t} + \frac{1}{2}g^*(t)e^{-j\omega_c t}, \quad \&
\]

\[
V(f) = F[v(t)] = \frac{1}{2}F[g(t)e^{j\omega_c t}] + \frac{1}{2}F[g^*(t)e^{-j\omega_c t}] = \frac{1}{2}[G(f - f_c) + G^*(-f - f_c)]
\]

where we used the fact that \( F[g^*(t)] = G^*(-f) \)

Note: \( \text{Re}\{a+jb\} = \frac{(a+jb)}{2} + \frac{(a-jb)}{2} = a \)
The total average normalized power of bandpass waveform $v(t)$ is

$$P_v = \langle v^2(t) \rangle = \int_{-\infty}^{\infty} \mathcal{P}_v(f) \, df = R_v(0) = \frac{1}{2} \langle |g(t)|^2 \rangle$$
Example: Spectrum of Amplitude Modulated Signal

AM Modulation

Evaluate the magnitude spectrum for an AM signal with the complex envelope $g[m(t)] = A_c [1 + m(t)]$.

**Solution:** The spectrum of complex envelope is $G(f) = A_c \delta(f) + A_c M(f)$

$$s(t) = \text{Re} \{ g(t) e^{j\omega_c t} \} = A_c [1 + m(t)] \cos \omega_c t$$

$$S(f) = \frac{A_c}{2} \left[ \delta(f - f_c) + M(f - f_c) + \delta(f + f_c) + M(f + f_c) \right]$$

where because $m(t)$ is real, $M^*(f) = M(-f)$ & $\delta(f) = \delta(-f)$ is even.

$$|S(f)| = \begin{cases} \frac{A_c}{2} \delta(f - f_c) + \frac{A_c}{2} |M(f - f_c)|, f > 0 \\ \frac{A_c}{2} \delta(f + f_c) + \frac{A_c}{2} |M(-f - f_c)|, f < 0 \end{cases}$$

- The 1 in $g(t) = A_c [1 + m(t)]$ causes extra delta functions to occur in spectrum at $f = \pm f_c$. 
Example: Spectrum of Amplitude Modulated Signal

- Total average signal power
  \[ P_s(f) = \frac{1}{2} A_c^2 \langle |1 + m(t)|^2 \rangle = \frac{1}{2} A_c^2 \langle 1 + 2m(t) + m^2(t) \rangle \]
  \[ = \frac{1}{2} A_c^2 [1 + 2\langle m(t) \rangle + \langle m^2(t) \rangle] \]

- If we assume that DC value of modulation is zero, then \( \langle m(t) \rangle = 0 \).

  Average signal Power = \( P_s(f) = \frac{1}{2} A_c^2 [1 + P_m] \)

  Power in the modulation \( m(t) = P_m = \langle m^2(t) \rangle \)

  Carrier Power = \( \frac{1}{2} A_c^2 \)

  Power in the sidebands of \( s(t) = \frac{1}{2} A_c^2 P_m \)

1) Note that \( s(t) = \text{Re}\{g(t)e^{j\omega_c t}\} = A_c[1 + m(t)]\cos \omega_c t \)
2) \( \langle v(t)^2 \rangle \) for periodic sinusoidal functions is \( \frac{1}{2} \)
3) \( P_m = \langle m(t)^2 \rangle \)
Amplitude Modulation

Evaluate the magnitude spectrum for an AM signal with the complex envelope $g[m(t)] = A_c[1+m(t)]$.

Assume:

$$m(t) = \cos(w_a t)$$

$g[m(t)] = g(t) = 1 + m(t)$

$s(t) = \text{Re}\{g(t)e^{jw_c t}\}$

$= g(t) \cos(w_c t)$

$= [1 + m(t)] \cos(w_c t)$
How can we model a bandpass filter?

- $v_1(t)$ & $v_2(t)$ are input & output bandpass waveform, & $g_1(t)$ & $g_2(t)$ are their complex envelopes.
  
  \[ v_1(t) = \text{Re} \{ g_1(t) e^{j\omega t} \} \]
  
  \[ v_2(t) = \text{Re} \{ g_2(t) e^{j\omega t} \} \]

- $h(t)$ & $k(t)$ = bandpass filter impulse response & its complex envelope.

- $f_c$ = carrier freq.

We can show:

\[ \frac{1}{2} g_2(t) = \frac{1}{2} g_1(t) \ast \frac{1}{2} k(t) \]

\[ \frac{1}{2} G_2(f) = \frac{1}{2} G_1(f) \frac{1}{2} K(f) \]

Thus, any bandpass filter system may be described and analyzed by using an equivalent low-pass filter.

Input BP Signal

\[ H(f) = \frac{1}{2} K(f-f_c) + \frac{1}{2} K^*(-f-f_c) \]

Output BP Signal

Modeling the channel passing the bandpass signal:
Linear Distortion of Bandpass Signals (1)

- For distortionless transmission of bandpass signals the channel transfer function, $H(f) = |H(f)| e^{i\theta(f)}$, need to satisfy the requirements over the signal BW:
  1) $|H(f)| = A$ = a positive real constant
  2) $-\frac{1}{2\pi} \frac{d\theta(f)}{df} = T_g = \text{constant}$, where $\theta(f) = \angle H(f) = -2\pi f T_g + \theta_0$ & $T_g = \text{envelope(group) delay}$

\[
\theta(f_c) = -\omega_c T_g + \theta_0 = -2\pi f_c T_d \quad \text{Td is Phase delay!}
\]

\[
H(f) = Ae^{j(-2\pi f T_g + \theta_0)} = Ae^{j\theta_0} e^{-j2\pi f T_g}
\]

We have A because the transfer function must be constant!
Linear Distortion of Bandpass Signals (2)

\[ H(f) = Ae^{j(-2\pi ft + \theta_0)} = Ae^{j\theta_0 e^{-j2\pi ft}} \]

Thus, we can represent the input as:

\[ v_1(t) = x(t) \cos \omega_c t - y(t) \sin \omega_c t \]

Thus, the output \( v_2 \) will be:

\[ v_2(t) = Ax(t - T_g) \cos[\omega_c(t - T_g) + \theta_0] - Ay(t - T_g) \sin[\omega_c(t - T_g) + \theta_0] \]

\[ \theta(f_c) = -\omega_c T_g + \theta_0 = -2\pi f_c T_d \]

Modulation (envelope) on the carrier (\( x \) & \( y \) components) is delayed by \( T_g \) (group delay) and carrier is delayed by \( T_d \) (phase delay).

Group delay & Phase delay!
Bandpass Sampling Theorem

• If a (real) bandpass waveform has a nonzero spectrum only over the frequency interval $f_1 < |f| < f_2$, where the transmission bandwidth $B_T = f_2 - f_1$, then the waveform may be reproduced from sample values if the sampling rate is

$$f_s \geq 2B_T$$

• Bandpass signal $B_T$ Hz wide can be represented over a $T_0$-second interval, provided that at least $N = 2B_T \cdot T_0$ samples are used
Example of Linear Distortion

- Assume we have an RC LPF with time constant (RC) of $10^{-5}$ sec, carrier frequency of 15 KHz, and bandpass signal BW of 1 KHz.
  - Find $H(w)$ and $|H(w)|$
  - Find 3dB BW (cutoff frequency)
  - Find the Phase Delay of the System
  - Find the Group Delay of the System at $f_c$
  - Does this system Distort the input bandpass signal?

Remember:

$$v_2(t) = Ax(t - T_g) \cos[\omega_c(t - T_d)] - Ay(t - T_g) \sin[\omega_c(t - T_d)]$$
Example of Linear Distortion

```matlab
% Select values for R and C.
N = 12;
R = 10e3;
C = 10e-9;
tau = R*C;
fo = 1/(2*pi*R*C);
j = sqrt(-1);

% Evaluate the Transfer Function
for (k = 1:N)
f(k) = 1000*2^(-10)*2^k;
H(k) = 1/(1 + 2*pi*f(k)*tau*j);
end;

% Plot the Transfer Function
B = log10(H);
HdB = 20*real(B);
Theta = 180/pi*imag(B);

% Second, plot the Time Delay function as given by Eq. (2-155).
% Evaluate the Time Delay response
for (k = 1:N)
Td(k) = (1/(2*pi*f(k)))*tanh(f(k)/fo);
end

% Evaluate the Group Delay response
% Note: From Eq.(2-154), Theta(f)=-arctan(f/fo) and
% using d/df arctan(x)/dx=1/(1+x^2) we get the Tg(f) as shown below.
for (k = 1:N)
Tg(k) = (1/(2*pi))*fo/((fo^2+f(k)^2);
end
```
Example of Linear Distortion
References

• Leon W. Couch II, Digital and Analog Communication Systems, 8th edition, Pearson / Prentice, Chapter 1

• Electronic Communications System: Fundamentals Through Advanced, Fifth Edition by Wayne Tomasi – Chapter 2