Sample Exam 2 Problems – Math 107

**Note.** This is not a template. It is likely that topics that don’t appear here will appear on the in-class exam. However, these problems are intended to give you an idea of the style and difficulty level of exam questions, as well as to remind you of some of the major topics we have covered.

1. The population of a hopping Midwestern city that starts with 25,000 people in 1980 grows by 20% over a 4-year period.
   (a) Find a formula for the population of the city as a function \( t \), the number of years after 1980.
   (b) Find the annual and continuous growth rates of the city. Make it clear which answer is which.
   (c) In what year does the population of the city reach 40,000?
2. Find the half-life of a radioactive substance that decays by 5% annually.
3. Simplify each of the following expressions as much as possible, writing your final answer without logarithms IF POSSIBLE.
   (a) \( e^{2 \ln(x + 2)} \)
   (b) \( \log(0.01^x) \)
   (c) \( \log(\sqrt{10} \cdot 10^x) \)
   (d) \( xe^{\ln x + \ln y} \)
   (e) \( \ln(e^{2x} + e^{2x}) \)
4. Below, you are given the graph of a function \( y = f(x) \) and three transformations of \( f(x) \). Find a formula for each of the three transformations in terms of the function \( f(x) \).

   ![Graph of y = f(x)](image1)

   ![Image of y = f(x)](image2)

   ![Graph of y = f(x)](image3)

   ![Image of y = f(x)](image4)

5. Find exact values for each of the following. To simulate a testing situation, try to do these without peeking.
   (a) \( \sin(7\pi/6) \)
   (b) \( \cos(3\pi/4) \)
   (c) \( \tan(5\pi/6) \)
   (d) \( \sec(11\pi/6) \)
   (e) \( \cot(\pi/2) \)
   (f) \( \csc(5\pi/4) \)
   (g) \( \sin \theta \) given that \( \cos \theta = 2/3 \) and \( \theta \) is in the 4th quadrant.

Turn Over →
6. Given to the right is the graph of a quadratic function. Use the graph and the fact that this graph goes through the point \((3, 28)\) to find a formula for the function. Then, find the **exact coordinates** of the vertex and the \(y\)-intercept of the graph.

7. Write each of the following quadratic functions in vertex form and then write down the vertex.
   
   (a) \(y = x^2 - 8x + 3\)  
   (b) \(y = 3x^2 + 33x + 1\)  
   (c) \(y = -2x^2 + 5x - 2\)  
   (d) \(y = x^2 + bx + c\)

8. In riding the Brown County ferris wheel, you board the wheel on a platform that is 3 feet above the ground and reach a maximum height of 53 feet above the ground. Assuming that the wheel rotates at a constant rate, and that it takes you 60 seconds to complete 2 complete revolutions, complete the following.

   (a) Sketch a rough graph of your height, \(h(t)\), above the ground as a function of time, assuming that you board the ferris wheel at time \(t = 0\). What is the amplitude, midline, and period of your function \(h(t)\), and what do they mean practically in the context of this problem?

   (b) Find a formula for the function \(h(t)\).

   (c) Use your formula to find your height after you have been on the ride for 45 seconds.