accommodation. Logical operations are means of organizing experience (schemas) that are superior to prior organization.

According to Piaget, an operation always has four characteristics: (1) It is an action that can be internalized or carried out in thought as well as materially; (2) it is reversible; (3) it always supposes some conservation, some invariance; and (4) it never exists alone but is always related to a system of operations (Piaget 1970a, pp. 21–22). Operations become truly logical during the concrete operational stage. Previous operations (at the preoperational stage) were prelogical, never meeting all the above criteria. One logical operation, already discussed, is reversibility. Two other structures central to concrete operations are *seriation* and *classification* (Piaget 1977a). These two logical operations are basic to the child's understanding of number concepts (Wadsworth 1978; Gallagher and Reid 1981).

**Seriation: Ordering Objects According to Differences**

Seriation is the ability to mentally arrange a set of elements accurately according to increasing or decreasing size, weight, or volume. The ability to seriate *length* develops throughout the preoperational and concrete operational stages. The task usually used by Piagetians to assess knowledge of seriation of length is a simple one. A child is presented with a set of approximately 10 sticks varying in length by small but perceptible differences (1/4 inch). The child is asked to order the sticks from the smallest to the largest. The examiner may show a properly arranged construction prior to asking the child to make his or her construction. Piaget's research discerns five levels in development of seriation of length knowledge.

At the first level, age 4 or earlier, children typically place some of the sticks in a construction, with no order discernible. At Level 2, children construct pairs comprised of a small stick and a large stick, but their constructions show no relationship between pairs. Any stick can be placed...
in relation to any other stick, but is not simultaneously related to 2 sticks. Four- and five-year-olds eventually begin to form groups of 3 sticks, but without any order between groups of sticks.

At the next level, which is transitional (between Level 2 and Level 3), advances are made and several partial coordinations are seen. Children age 5–7 years frequently align the tops of the sticks (as in Figure 5.2) while paying no attention to the alignment of the bottoms of the sticks. Some children successfully order 4 or 5 sticks in a set, but usually no more.

At Levels 3 and 4, children age 7–8 successfully order the set of 10 sticks (as in the model), but there are qualitative differences in method between the two levels. Level 3 children typically use a trial and error approach.

The entire series is finally ordered but by an empirical grouping method, that is, with local errors and corrections afterwards. On the other hand, the subject has not mastered the transitivit y problem. (Piaget 1977, p. 131)

At Level 3, the child is unable to order 3 or more sticks in order mentally, demonstrating a lack of **transitivity**. If it is unclear whether a child can order the series mentally, he or she can be asked to place the sticks successively in order behind a visual screen. This requires a mental ordering of the series for successful construction.

At Level 4, children have no difficulty with the seriation task. The 10 sticks are ordered accurately without trial

<table>
<thead>
<tr>
<th>no order</th>
<th>ordered pairs only</th>
<th>transitional</th>
<th>accurate ordering</th>
</tr>
</thead>
<tbody>
<tr>
<td>level 1</td>
<td>level 2</td>
<td>level 2-3</td>
<td>levels 3 and 4</td>
</tr>
<tr>
<td>&lt;4 years</td>
<td>4-5 years</td>
<td>5-7 years</td>
<td>7-8 years</td>
</tr>
</tbody>
</table>

**FIGURE 5.2**
and error. Children employ strategies such as searching for the smallest stick, then the next smallest, and so on.

This strategy implies both transitivity and the reversibility inherent in an operational structure: any stick is longer than all the proceeding ones and also shorter than all those that follow it in the series.

(Gallagher and Reid 1981, p. 97)

Children at this level also have no difficulty with the visual blind problem. They are confident that their constructions are correct, even when they cannot see them.

Children's knowledge about seriation is constructed over a period of years. Each advance is a new equilibrium in the child's reasoning. Seriation of length is generally attained around age 7 or 8.6

**Classification: Mentally Grouping Objects According to Similarities**

In Piaget's traditional classification studies, children are presented with sets of objects (such as geometric shapes varying in size and color) and are asked to put the objects that are alike together (Piaget and Inhelder 1969; Piaget 1972b). Three levels of development emerge from these studies.

**Level 1.** Children of four or five years of age typically proceed by selecting objects to go together based on similarities. But the criterion they use is what is similar between two objects at a time. Thus, a child may put together a black circle and a white circle (both circles), then may add a white triangle to the white circle (both white), then may place a gray triangle with the white triangle (both triangles), insisting that they all go together (Figure 5.3). Objects are assimilated to similarities between individual pairs of objects only. Differences between objects in the set are ignored. There is no plan for the total set.

**Level 2.** Through age 7, children typically form collections of like objects along one dimension. That is, circles